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SPACE NAVIGATION HANDBOOK

NAVPERS 92988

PREFACE

This Space Navigation Handbook has been prepared at the United States Naval Academy by the pilot class in space navigation, convened by the Chief of Naval Personnel on 10 July 1961. Though originally intended only to introduce a simplified concept of space navigation restricted to the Earth-Moon system and requiring a minimum of equipment, the manuscript has been extended and enlarged so as to include necessary background information. In its present form it is intended as a textbook for a broad introductory course in space navigation presented at the graduate level and covering about a four-week period.

By no means a definitive work, the Handbook presents some well-known problems of astronomy and physics in a highly condensed fashion, and covers little of the present technology of space flight. It does, however, provide a self-contained explanation for a concept, the development of which was the primary objective of the pilot class. References will be found at the end of many of the chapters, should the reader want to find more background information, or investigate a particular point in more detail. Items of only general interest have been relegated to the Appendices.

The Handbook is published under the direction of the Chief of Naval Personnel, and valuable support has been given the writers by many activities within the Naval Establishment. Preliminary research by Captain P. V. H. Weems, USN (Ret.) prior to the official establishment of the space navigation class was supported by a grant from the American Philosophical Society. Captain Weems was recalled to active duty to conduct the space navigation class. Members of the class were Ensign Paul D. Bowman, USN, Ensign Gene M. Cunningham, USN, Ensign Rod L. Mayer, USN, Ensign G. Daniel Zally, USN.

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CHAPTER I

INTRODUCTION

Within the past decade we have entered a new age and have developed a new technology, for man has made his initial ventures into our last great frontier, space. Once relegated to the world of dreams and science fiction, space flight has become a *fait accompli*. As more ambitious goals are set for manned flight, the question of navigation in space takes on increasing importance. The success of any assigned mission and even the safety of the astronaut are doomed unless the man in the space vehicle can determine his position with respect to some reference frame and predict future positions. It is with this thought in mind that we take up the subject of space navigation.

101. THE CHARACTERISTICS OF SPACE NAVIGATION

The advent of the space age calls for new methods of navigation, just as man's conquest of the sea and the air required the development of fast, reliable navigational techniques. The inherent differences of space travel, however, place certain limitations on a navigation method. Travel is no longer in straight lines, but in orbits; speeds are measured not in tens or hundreds of knots, but in tens of thousands of knots; a natural horizon as a base for measurement of celestial altitudes is no longer available; direction of motion is no longer controllable to the same extent as in sea and air travel. And, because of the extremely high magnitudes of velocity and distance, a positional fix accurate to within a few miles calls for a relative accuracy unknown (and unnecessary) in Earth-bound navigation.

Faced with these new conditions, what will be the course of space navigation? Is it a problem solvable only by electronic instruments and computers independent of all human functioning?

Because of the limited reliability of present electronic equipment, space navigation must never become completely divorced from the activities of the human navigator. The "mean time to failure" of the components in a complicated electronic system is measured in terms of hours or days. This is perfectly adequate for modern aircraft, which find maintenance and repair facilities readily available. The reliability expectancy of space craft navigation equipment must be, however, in the order of months or years. There is a high price to be paid for equipment failure in space!

It is for this reason that a manually operated self-contained navigational system requiring no external power supply must be included in every manned space vehicle. This does not mean that electronic equipment should not be used, for it undoubtedly will be necessary for certain complicated operations requiring close tolerances and critical timing. But a system must be available that would allow a crew to at least return to the Earth's surface in the event of an emergency which disabled electronic equipment. The general specifications for such a system are as follows:

1. External power sources are not required,
2. Rigid attitude control is not required,
3. Breaks in the craft hull for observational purposes are minimized,
4. The equipment is simple to operate and maintain,
5. Extensive calculations are not necessary,
6. Relatively fast position determination is possible, and
7. The equipment is light weight and small in size.

102. OUTLINE OF THE HANDBOOK

The purpose of the Handbook is twofold. First, we shall present a concept of space navigation which lends itself to such a system as described above. Second, we shall cover in an introductory manner some of the material that would form a solid background for a prospective student of space navigation.

This introductory chapter is followed by a discussion of the near-Earth space environment,

i.e., the Earth-Moon system (chapter 2). We then examine some of the physical principles that are applicable to satellite motion near the Earth, using a central force approximation (chapter 3). A more detailed inspection of the orbital parameters and their determination (chapter 4) and a review of coordinate systems (chapter 5) set the stage for a discussion of satellite tracking techniques both from the Earth (chapter 6) and from the satellite (chapter 7). Position prediction in space is then covered (chapter 8), and a proposed Space Almanac is introduced (chapter 9). The Handbook concludes with an inspection of the problems of rendezvous in space (chapter 10) and reentry to the Earth (chapter 11). Items of a general interest are included in the several Appendices.

103. DEFINITIONS

As in any field, if we are to proceed intelligently, then we must first define the specialized technical terms associated with our subject. The following is a group of basic definitions from celestial navigation that we shall use throughout the Handbook. A more complete list may be found in the *American Practical Navigator*, H.O. Pub. No. 9.

These definitions serve primarily as a review. If, of course, the reader is familiar with the terms, he may bypass this section.

A sphere is a body bounded by a surface all points of which are equidistant from a point within, called the center. In locating positions on the Earth, it is convenient to consider the Earth to be a sphere. A spherical coordinate system whose origin lies at the center of the Earth is then used to fix positions. Since the radius will be the same for any point on the Earth's surface, two angular coordinates will determine a point uniquely.

A great circle is the intersection of the surface of the sphere with a plane passing through its center. An important theorem of solid geometry states that the shortest path between two points on the sphere is the minor arc of a great circle containing the points.

The axis of the Earth is that line about which it rotates (or, more precisely, a line through the center in the direction of the angular momentum vector). The axis pierces the Earth at two points, called the poles. One pole (that in the direction of the angular momentum vector) is labeled north, the other south.

The equator is that great circle whose plane is perpendicular to the axis of the Earth. All points on the equator are 90° from the poles.

Meridians are great circles of the Earth which pass through the two poles. The plane of every meridian contains the Earth's axis, and the axis divides the meridian into two equal parts.

The prime meridian is that meridian which passes through the Royal Observatory at Greenwich, England. It is used as a reference meridian.

The longitude of a point is the central angle between the meridian through the point and the prime meridian. It is measured eastward or westward from the prime meridian through 180°, and is labeled E or W to indicate the direction of measurement.

Parallels are the intersections of the Earth's surface with planes parallel to the plane of the equator.

The latitude of a point is the central angle between the parallel through the point and the equator. Latitude is measured northward or southward from the equator through 90°, and is labeled N or S to indicate the direction of measurement.

The celestial sphere — Whenever we look into the night sky, we see the stars as points on a vast dome. Some stars are brighter than others, but all of them are so tremendously far from the Earth that they appear to be on the surface of an enormous hollow sphere. This we call the celestial sphere. The radius of the celestial sphere is essentially infinite, and the Earth is just a point at its center. Points on the Earth's surface may be projected from the center of the Earth onto the celestial sphere, as may the equator, meridians, and parallels.

The vertical at a point is the direction parallel to the direction of the gravitational field. For all practical purposes, the vertical is an extension of the line through the point and the center of the Earth.

The zenith (Z) of an observer (O) is that point on the celestial sphere vertically overhead.

The zenith distance of any point is the central angle between rays from the Earth's center passing through the point and through the zenith.

The nadir (Na) is the point on the celestial sphere directly beneath the observer, or the point diametrically opposite the zenith.

The celestial poles are the projections of the north and south poles of the Earth on the celestial sphere; they are given corresponding names. To an observer in northern latitudes the north celestial pole is above the horizon, while the south celestial pole is below it.

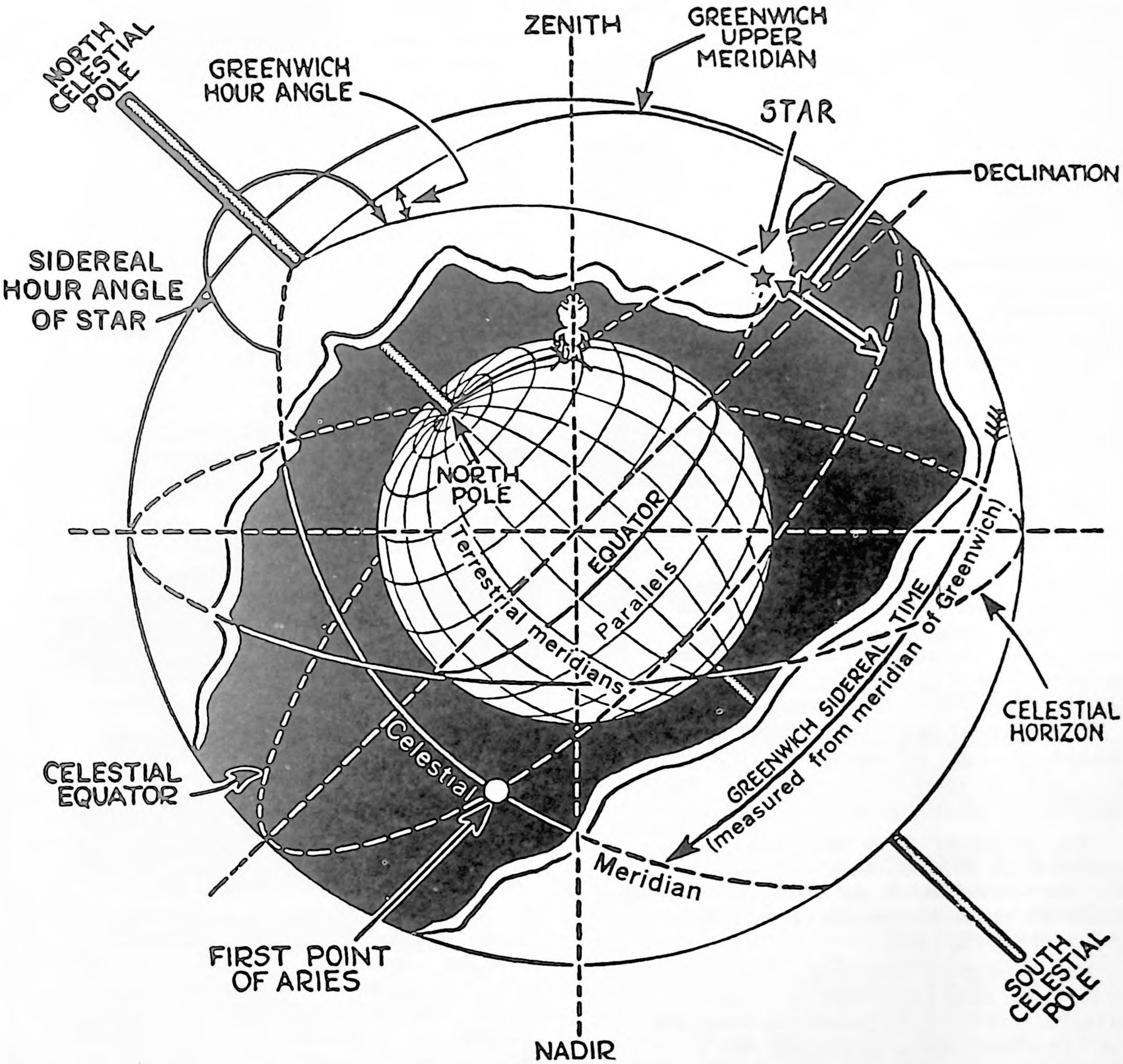


Figure 103A.–The Earth and the celestial sphere. This drawing shows the relationship between the celestial and terrestrial coordinate systems. The observer is on the Greenwich meridian, so that the GHA is also the LHA.

The elevated pole is that pole with the same name as the observer's latitude; it is the pole which is "elevated" above the observer's horizon.

The celestial equator, or equinoctial, is the great circle on the celestial sphere formed by the intersection of the Earth's equatorial plane. All points on the celestial equator are 90° from the celestial poles.

It is noteworthy to point out that every point on the Earth has a corresponding point on the celestial sphere. This point is found by projecting the terrestrial point out to the celestial sphere along a line through the center of the Earth.

A celestial meridian is a great circle on the celestial sphere passing through the poles. The celestial meridian of an observer is the

intersection of the plane of his terrestrial meridian with the celestial sphere.

Celestial parallels are the intersections of the celestial sphere with planes parallel to the plane of the celestial equator.

The declination of a point on the celestial sphere is the central angle between the celestial parallel through the point and the celestial equator. It is measured in the same way as terrestrial latitude.

An hour circle is a great circle of the celestial sphere passing through the poles and some celestial body or point. It differs from a celestial meridian in that it is considered fixed in space, while a celestial meridian is considered to rotate with the Earth.

The ecliptic is the great circle of the celestial sphere that represents the apparent path of the Sun around the Earth due to the Earth's annual rotation about the Sun. The plane of the ecliptic is inclined 23°442 to the plane of the equinoctial.

The vernal equinox is the intersection of the ecliptic and the equinoctial where the declination of the Sun changes from south to north. This point, fixed in space relative to the stars, is called the First Point of Aries, or simple Aries, and is designated by the symbol ♈.

The sidereal hour angle (SHA) of a point is the central angle between the hour circle of the point and the hour circle of Aries, measured westward from Aries through 360°.

The local hour angle (LHA) of a point is the central angle between the hour circle of the point and the meridian of an observer, measured westward from the celestial meridian of the observer through 360°.

The Greenwich hour angle (GHA) of a point is the local hour angle with the observer on the prime meridian. It is measured westward from the Greenwich meridian through 360°.

The right ascension (RA) of any point on the celestial sphere is the central angle between the hour circle of Aries and the hour circle of the point, measured eastward from Aries. Right ascension is normally measured in units of time, with 24 hours equivalent to 360°.

Greenwich sidereal time is the GHA of Aries, measured in terms of time rather than arc. ($1^h 00^m 00^s$ = 15°00)

Figures 103A and 103B help demonstrate the preceding definition.

Other definitions will be given elsewhere in the Handbook when and where they are used.

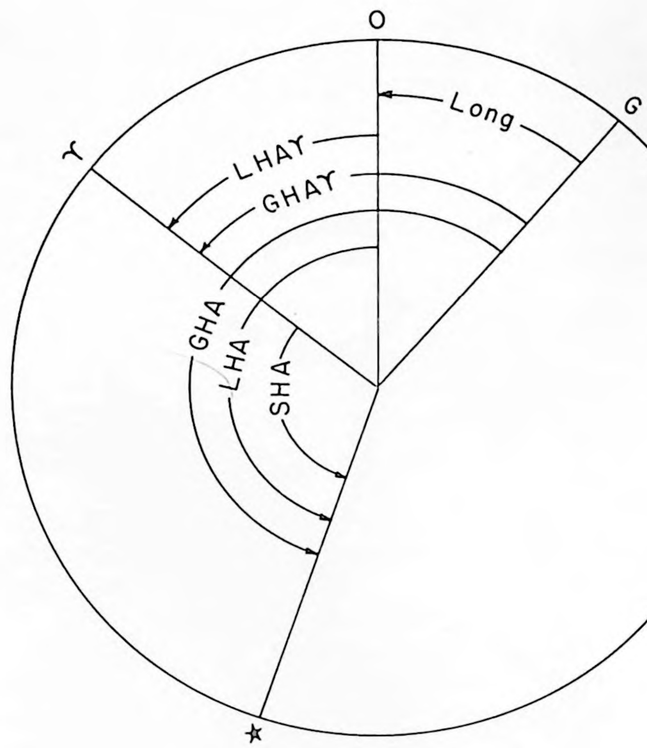


Figure 103B.—The relationship between longitude and hour angle. Westward directions are measured counterclockwise. G is the Greenwich meridian, O the observer's meridian. ♈ is the hour circle of Aries, and ☆ is the hour circle of a star.

104. CELESTIAL AND TERRESTRIAL COORDINATES

Since space flights will originate on the Earth and will eventually terminate there, it is convenient, at least for near-Earth navigation, to give positions relative to the Earth's center. Since the Earth is nearly a sphere, and since marine and air navigators are familiar with spherical coordinate systems, we shall use spherical coordinates to determine positions in space. Two such systems are in common navigational use. One system, which rotates with the Earth and is useful for locating positions on the Earth, uses the angular coordinates latitude and longitude. The other, which is fixed in space relative to the star field, gives positions in terms of declination and sidereal hour angle.

For most practical purposes, the space navigator will not be interested in knowing the location of the point on the Earth directly beneath him so much as in knowing his position in space relative to the Earth's center. During launching and reentry, however, and perhaps for special operations, he may need to know the location of his geographic position (GP). In some cases an observer in a space vehicle will be able to identify a terrestrial landmark directly beneath him.

This would allow him to find his GP instantly, without the need of a coordinate transformation.

Still the space navigator will need to be on familiar terms with sidereal hour angle (SHA) and declination, and the relation of these two celestial coordinates to the corresponding terrestrial coordinates, longitude and latitude. Since latitude is measured from the equator along meridians to the poles, and since declination is measured from the equinoctial (celestial equator) along celestial meridians to the celestial poles in the same units, the values of the latitude of a point and the declination of its zenith are the same. The relation of longitude to SHA depends upon the orientation of the Earth, which is a function of time. If the value of the GHA of Aries is known at a given time (it can be found in the almanacs), simple addition is all that is required to convert SHA to longitude, and vice versa.

105. UNITS

There are many systems of units, and it is unfortunate that a single, uniform set has not been adopted for use by everyone. Distances can be given in feet, statute miles, nautical miles, or kilometers. Speeds are measured in knots, miles per hour, feet per second, or kilometers per second. This lack of conformity adds to the navigator's burden. Since the official (legal) units for sea and air navigation are the nautical mile and the knot, we shall use them where applicable. In addition, the nautical mile is a natural unit, being approximately the length of a 1' arc along a great circle on the Earth.

Likewise the familiar terms of latitude and longitude or their celestial counterparts, declination and sidereal hour angle, will be used in preference to coordinates based on other frames of reference. We feel that there is great advantage in selecting terms in common use among navigators. Angular coordinates will be given in decimal fractions of degrees, rather than in degrees, minutes, and seconds of arc. This allows simplified tabulation and computation of angular quantities, and facilitates use of digital data processing systems where necessary.

106. THREE-DIMENSION FIXES

A ship at sea is considered to be moving in a plane, and motion is restricted to two dimensions. Its position may be given on a chart in terms of latitude and longitude. Its speed would be shown by the pitometer log, and its direction of movement by a compass. But if a man is shot up into space, he is operating in three-dimensions. His position is given as an altitude above the Earth in a specific direction from its center. For example, a space vehicle may be 10,840 miles from the center of the Earth in a direction with declination N 20°17, SHA 240°76. These three coordinates fix the position of the vehicle in space relative to the Earth.

It is this three-dimensional aspect of space flight that requires new methods of navigation. The basic principles of celestial navigation still hold, but they must be applied in different ways. The determination of the third coordinate, the distance from the Earth, will be accomplished by new techniques.

Another problem arises in the prediction of future positions. On the Earth, motion in two dimensions allows the navigator to dead reckon to find his position after a period of time. But the astronaut does not move in a straight line nor at a constant speed. His direction of motion is constantly changing. The terrestrial methods of position prediction by linear plots has no simple analog in space. Again, new methods are needed.

107. METHODS FOR SPACE NAVIGATION

In this section we shall introduce a method of space navigation which will be covered at length in chapter 7. We shall restrict ourselves to flights that will go out as far as the Moon, i.e., about 220,000 nautical miles from the Earth. We further limit ourselves to a minimum of equipment. Since every pound of payload that is put into orbit requires up to a thousand pounds of rocket fuel for the launch vehicle, this is evidently a very sensible consideration. We also desire a system that maximizes reliability. The use of complicated sensors or elaborate computing equipment, besides adding to the dead weight of instrumentation, presents the possibility of failure. In essence we want a system, both simple and dependable, that will allow us to determine our three-dimensional position in space with a reasonable degree of accuracy.

Since the space observer can <u>see</u> the Earth's position relative to his own, but cannot see his own position relative to the Earth, we propose that he observe the motion of the Earth as it <u>appears</u> to orbit about him. This is analogous to the fact that, although the Earth actually orbits about the Sun, the Sun appears to move around the Earth. The basic principles upon which cislunar space navigation will be based are quite simple and may be briefly stated as follows:

1. Determine SHA and declination by optically observing the position of the Earth's center against the star field.

2. Determine distance stadimetrically by measuring the angle subtended by the Earth's disk.

Since the essential operation of space navigation will be done optically and with a minimum of aids, it should be nearly instantaneous. The direct viewing of the Earth's center against the star field gives the navigator's geographic position (GP) in terms of sidereal hour angle and declination, based on either personal knowledge of the stars or reference to a star chart. The distance from the Earth is found by viewing the Earth through some device featuring range circles etched on a transparent sheet or star globe, or the Earth's apparent angular diameter may be measured by a marine sextant, perhaps modified in some respects. In this latter technique, a table could correlate range and angular diameter, or the sextant could be marked to read range directly.

The important point to make is that when an astronaut views the center of the Earth, he thereby establishes his vertical. Should his zenith coincide with a star, it would only be necessary to pick from an almanac the star's position for that instant, thereby determining his position exactly. It is essential that the reader understand the significance of the observer's vertical, zenith, nadir, and star positions, and how they may be used to guide astronauts safely through space.

This concept of space navigation will be developed in detail later in the Handbook, and apparatus employing its principles will be described. Methods for position prediction will be advanced, as well as a Space Almanac.

CHAPTER 2

THE SPACE ENVIRONMENT

As have the mariner and airman, the astronaut will put to practical use his knowledge of the heavens. He will be even more dependent on celestial bodies to guide him in space. While there is a lot we do not know about the visible bodies in the heavens, there is a great deal that we do know in spite of the fact that no human being has been out there to verify our facts.

By studying figure 200 the student will, by giving free rein to his imagination, realize that the Earth, and even the Solar System itself, may be considered mere points in the universe. These facts should give us a wholesome respect for the Science of Astronomy, and for astronomers.

201. THE ASTRONOMERS' UNIVERSE

Our spiral galaxy, the Milky Way, is just one of millions of other galaxies which are distributed throughout space, receding from us in all directions as far as our largest telescopes can see. Their radial velocities increase with increasing distance from us as measured by the redshift in their spectra. Those at the outer limit of the 200-inch telescope approach one third the velocity of light.

The Milky Way, composed of 100 billion stars, interstellar gas, and dust has what is called a spiral arms structure—a central spheroidal stellar region surrounded by a flat disk of stars 100,000 light years in diameter containing spiral arms of stars, gas and dust. Scattered around our galaxy are star clusters of varying sizes.

Inconspicuously placed, in one of the spiral arms, near the galactic equatorial plane, about 35,000 light years from the center is the Sun, an average type star, as far as stars go, which is of great importance to our existence and receives more study than any other astronomical body. The Sun is revolving about the center of the galaxy at about 600,000 knots. Her closest star neighbor is Alpha Centauri, a binary star 4.5 light years away. Accompanying her are a multitude of bodies ranging in size from microscopic meteors to the giant planet Jupiter. This group is called the solar system with the most distant planet, Pluto, being about 5.5 light hours away from the Sun. This distance is great when compared to the 8.3 light minutes the Earth is from the Sun and the 1.3 light seconds the Moon is from the Earth but very small when compared to the 4.5 light years to the nearest other star system and the 2×10^6 light years to a neighboring galaxy.

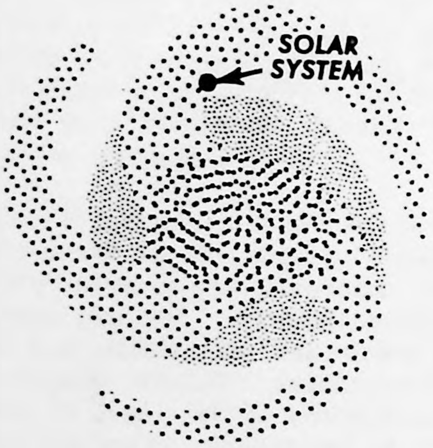
The motions of the 9 known principal planets and their satellites together with the thousands of minor planets, asteroids, are governed primarily by gravitational forces. They, therefore, describe paths which are predictable, and in most cases well calculated. With the exception of Pluto, the inclinations of the orbits of the principal planets to the ecliptic do not exceed 8°, nor are any of their eccentricities greater than 0.25.

Instead of discussing each of the planets let's look very briefly at the planet most likely to be visited. This is Mars. Because its atmosphere affords visual penetration, the surface of Mars has been observed and analyzed. It appears to be the only planet, besides Earth, which could support some form of life. Carbon dioxide is the main constituent of the atmosphere with small amounts of nitrogen. The carbon dioxide is broken up by ultraviolet radiation into carbon monoxide and atomic oxygen. The atmosphere of Mars will protect its surface from meteors and make high pressure space suits unnecessary.

Following is some physical data of Mars: mean distance from Sun, 123 million nautical miles; period of revolution, 686.980 days; eccentricity of orbit, 0.093; inclination to ecliptic, 1°51'; mean diameter, 3,650 nautical miles; mass, 0.11, that of the Earth; density, 4.12, that of water; period of rotation, $24^h\ 37^m$; inclination of equator to orbit, 24°; oblateness, 1/192. Since its axis is inclined 24° to its orbit Mars exhibits Seasons which accounts for the formation and disappearance of the thin ice caps of the Martian polar regions. The mean temperature

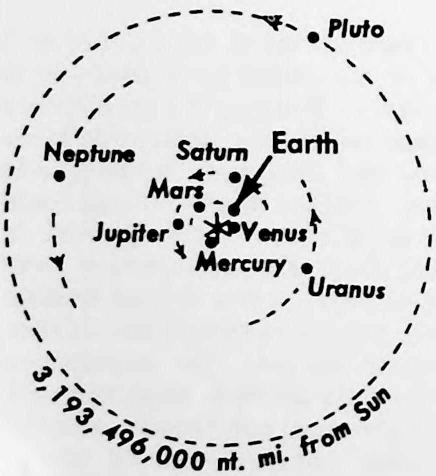
(1) THE UNIVERSE –

OBSERVABLE DISTANCE FROM OUR GALAXY IN ANY DIRECTION ABOUT 2 BILLION LIGHT YEARS



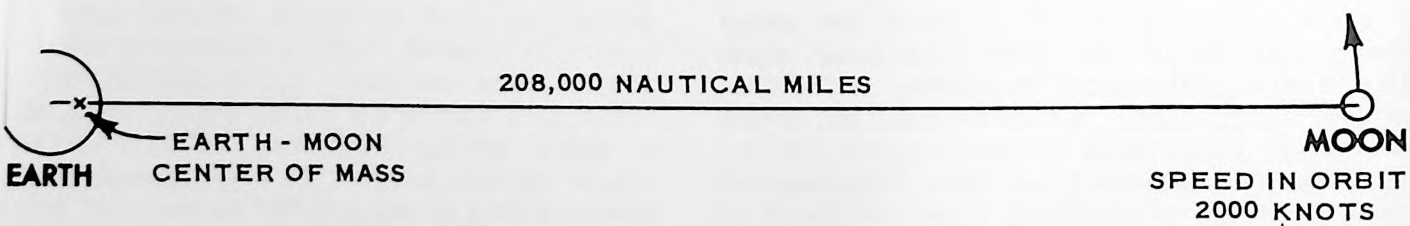
(2) OUR GALAXY (The Milky Way System) –

ESTIMATED AS 100,000 LIGHT YEARS IN DIAMETER; SUN IS APPROXIMATELY 35,000 LIGHT YEARS FROM CENTER



(3) SOLAR SYSTEM –

ORBIT OF PLUTO ABOUT 11 LIGHT HOURS IN DIAMETER. DISTANCE FROM SUN TO EARTH ABOUT 8.3 LIGHT MINUTES. EARTH MOVING AROUND SUN AT 58,000 KNOTS



NOTE: EARTH AND MOON SHOWN AT DOUBLE SIZE IN PROPORTION TO DISTANCE

Figure 200.

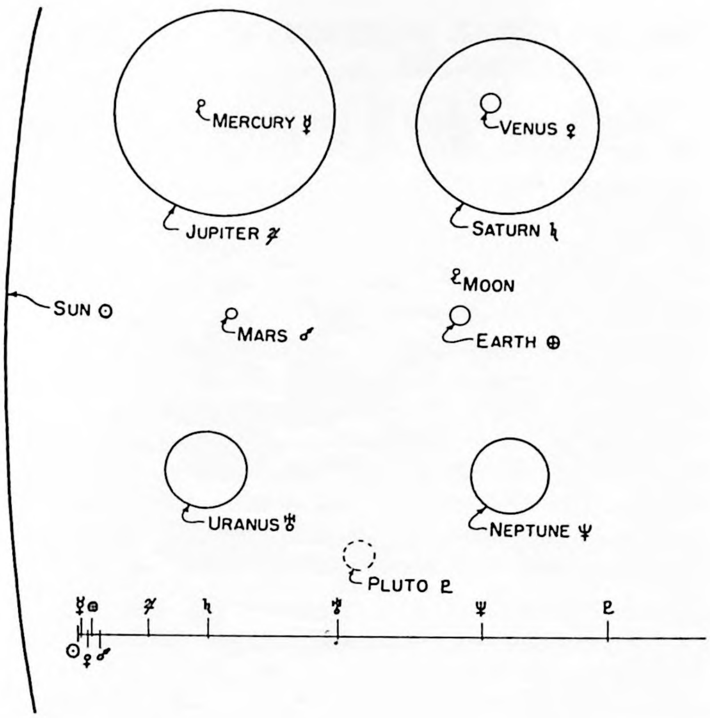


Figure 201.–Relative Distances and Sizes of the Planets.

for the planet is about -60°F, making it cold and dry by Earth standards.

Mars has two satellites. Phobos, the larger inner moon, is about 10 miles in diameter and revolves around Mars in only $7^h\ 39^m$ at a distance of 3,200 nautical miles from the surface. The other moon, Deimos, is 10,900 nautical miles from the surface of Mars and takes $30^h\ 18^m$ to complete a revolution. Both satellites have made it possible to determine the mass of Mars accurately by application of Kepler's third law (section 303). Their orbits lie very close to the equatorial plane of Mars and are almost circular.

The rest of the solar system is made up of gaseous material, meteoritic matter, comets, and high-energy charged particles. A magnetic field will capture the charged particles and produce radiation zones about the source, such as the Van Allen radiation belts around the Earth. These belts impose a hazard to electronic equipment and human life and greatly limit the choice of flight paths for a space ship.

202. THE EARTH-MOON SYSTEM

There is no doubt that the first destination of space voyages will be the Moon. It appears to us the largest body in the heavens and has, next to the Sun, the greatest influence on the Earth and its inhabitants.

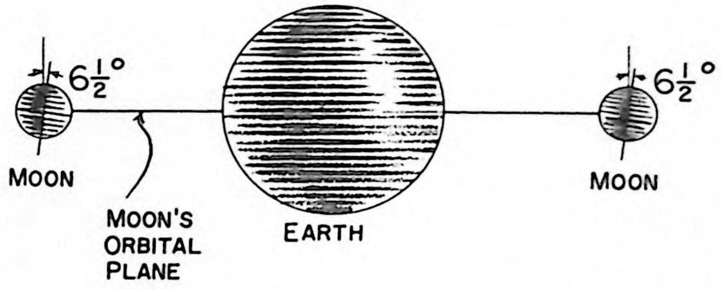


Figure 202A.–Libration in Latitude.

The mass of the Moon is about 1/81st of the Earth's mass. This makes the Moon the largest satellite relative to its primary in the solar system and is why the Earth-Moon System is sometimes referred to as a double planet. The Earth and Moon revolve about their common center of mass, located only 2,520 nautical miles from the Earth's center. This point, called the barycenter, in turn revolves about the Sun in similar fashion. The mean distance between the centers of the Earth and Moon is 208,000 nautical miles. The Moon rotates on its axis in the same period in which it revolves around the Earth, thus the Moon would present the same side toward the Earth all the time if it weren't for librations or apparent oscillations. These result principally because: (1) The Moon's equator is inclined 6-1/2° to the plane of its orbit causing it to tip toward and away from us so that we can see 6-1/2° over the north pole and 6-1/2° beyond the south pole (figure 202A); (2) The revolution in the Moon's elliptical orbit (eccentricity of 0.055) is not uniform, whereas the rotation on its axis is nearly uniform. Although the rotation and revolution come out together at the end of each month, they do not keep in step. We therefore see as much as 7-3/4° around each edge on account of this east-west rocking (figure 202B); (3) As a consequence of the Earth's rotation it is possible to see about 1° over the western and eastern edges at moonrise and moonset respectively (figure 202C). In addition to the above apparent oscillations the Moon does have a slight physical libration because its rate of rotation is not quite uniform. The final result is that instead of only seeing half the Moon's surface we see 59% of it. (Those more interested in the motion of the Moon see Appendix B.)

No satisfactory answer can be given as to the time and place of origin of the Moon. Whether or not the Moon was captured by the Earth or developed by the Earth splitting while still molten or was formed of condensed gas with the Earth, but separately is impossible to say.

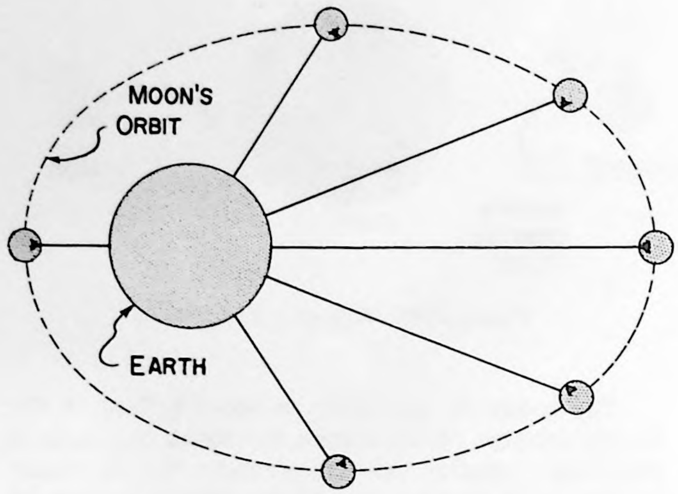


Figure 202B.–Libration in Longitude.

203. CELESTIAL POSITION, VELOCITY AND DISTANCE

Except for bodies of the Solar System, visible heavenly bodies might be considered to be at infinite distances. Suppose a space vehicle has two telescopes aligned on two stars and is moving in free flight without power, it would remain in that attitude and point to the same stars if it were moved bodily a hundred million miles in any direction. This is another way of saying distance means little so far as the appearance of the star field is concerned.

Direction and position of stars, unlike distance, may be measured and sensed. For example, the angle between two stars may be measured accurately, and this angular distance would appear to remain nearly fixed over long

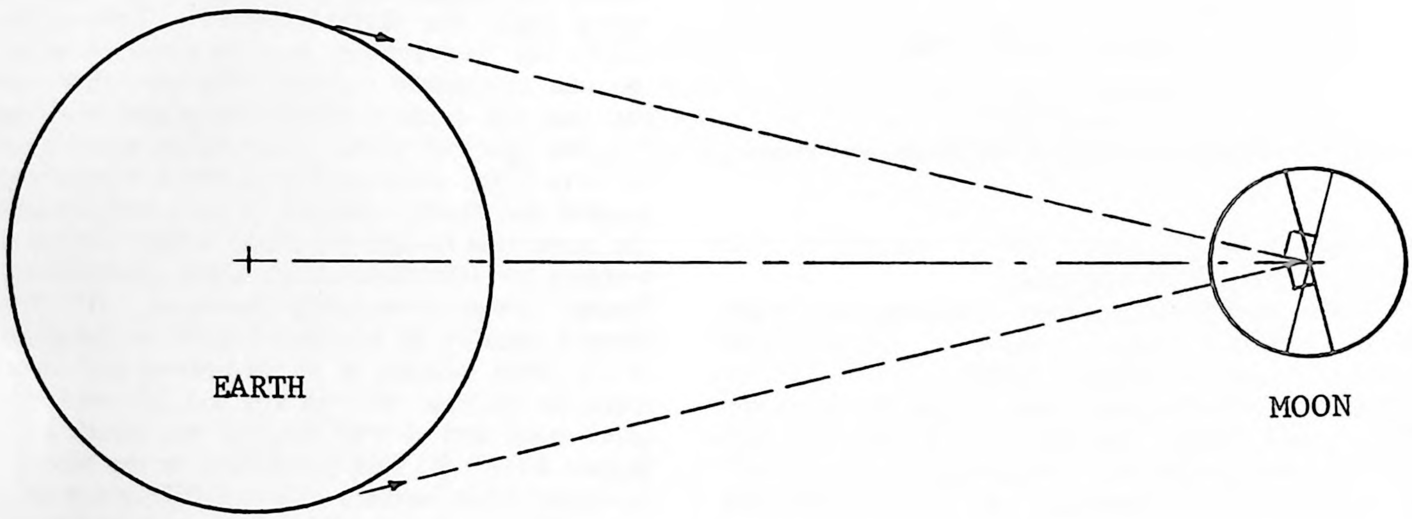


Figure 202C.–Diurnal Libration.

From the variance in lunar and terrestrial relief one would have cause to think that their histories have been somewhat different. However, an atmosphere, such as the Earth's, which the Moon happens not to be massive enough to attract, would account for many of the differences.

Is there life on the Moon? Temperatures range from 100°C at the equator at noon in sunlight to -100°C at midnight. There is no air or free water nor an atmosphere of any kind to filter intense radiations. Uninviting though it may be, the answer awaits men exploring it firsthand.

Probably one of the uses of a lunar base after man has conquered the environment will be an observatory, removed from the turbulent atmosphere of the Earth, to obtain valuable information about Mars.

periods of time, though we know stars have high velocities of their own. The stars are so far away that the great speeds at which they course through the heavens can scarcely be detected, and would have no practical significance for the astronaut.

To sum up, we know the positions and the directions of stars quite accurately. The distances of stars in the universe are so great that the relative distances in the Solar System would be insignificant, not to mention the still shorter distances of the Earth-Moon System. Therefore, the only distances of prime concern to the astronaut are the relative distances between his space vehicle and the Earth and the Moon.

204. STAR IDENTIFICATION

To say that star positions are known is an understatement. In spite of refraction and any

uneven earthly motion, astronomers can predict, for a given time, the point on the Earth's surface which will have a selected star in its zenith ten years in the future to an accuracy of about twenty feet.

Since star positions are known and may be used as "lighthouses" in the sky, it behooves the astronaut to become familiar with stars and their convenient configurations and magnitudes. In fact, the astronaut should memorize the positions and identity of as many stars as possible so they may be of the greatest and most immediate help when needed. Chapter 7 will explain how a knowledge of stars may be used for position finding.

205. EVOLUTION OF SPACE FLIGHT

The popular concept of space appears to underrate the importance of the Moon and the significance of the Earth itself in practical space navigation, or perhaps we should say in "space piloting." Furthermore, to some people the term "space" in itself seems to signify millions of miles, which a little thought will make clear that the first space voyages will not likely cover millions of miles, but more likely a few hundred, or at most a few thousand miles. The first astronauts were satisfied with making a short pioneering voyage into space and returning to Earth alive to relate their experiences.

It is predicted that humans will land on the Moon by 1970. The next goal, Mars or Venus most likely, will entail voyages of millions of miles. If it requires ten years to work up to flights of about 220,000 miles, it would appear that another ten years at least would be required to increase this range to the millions of miles required for planetary voyages. In addition, the optimal times for launching a vehicle so to obtain minimum fuel expenditure or minimum time of flight occur at rather infrequent intervals, this factor causing further delays for firing a manned space probe. Therefore, in the near future, the Moon and artificial satellites will most probably be our attainable goals.

It seems logical to assume that distances, speeds and time in transit will increase by evolution, but that astronauts will not soon blast off for long flights into outer space.

206. CISLUNAR NAVIGATION

For flights to the Moon, the Earth itself will not only be the most conspicuous body in the heavens, but it will also afford the simplest means for navigation, as will be explained in chapter 7. Distances from the Moon may be found quite simply by stadimetric methods. Direction from the Moon could be determined in terms of sidereal hour angle and declination. With high relative speeds and relatively short distances, the Moon's apparent position in the heavens will change rapidly. Since the direction of the Moon from the observer is exactly opposite the direction of the observer from the Moon, there might be times, in the vicinity of the Moon, where it would be advantageous to give the observer's position relative to the Moon. This is especially true if the space flight is to terminate with a lunar orbit or lunar landing. Since we know from the *Almanac* the Moon's hour angle and declination, as observed from the center of the Earth, we can, by fixing a position relative to the Moon, transpose this position to a position in space relative to the Earth by translating the origin of the coordinate system (chapter 5).

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CHAPTER 3

PHYSICS OF TWO-BODY SYSTEMS

The study of satellite motion involves, to the first approximation, nothing more than the principles which were developed by Newton. It is possible to neglect the effects of the Sun and Moon when considering close-in satellites of the Earth. In fact, because of the relative size of the Earth and satellite, the Earth's motion may be neglected, and the two-body problem reduced to a one-body problem in a central force field.

This introductory analysis will first inspect the circular orbit using Newton's Second Law of Motion, the law of universal gravitation, and expressions for kinetic and potential energy as basic building blocks. Later in the chapter, Kepler's laws of motion will be discussed, as well as the relationship of energy, angular momentum, and eccentricity of elliptical orbits.

301. GRAVITATIONAL FORCE

The force between any two particles having masses M and m, separated by a distance r, is an attraction acting along the line joining the particles, and has the magnitude

$$F = \frac{GMm}{r^2} \qquad (1)$$

where G is a universal constant having the same value for all pairs of particles. Note that the above G is <u>not</u> the acceleration due to gravity, commonly denoted by g. The value of G is determined by experiment and has the value in the mks system of

$$G = 6.673 \times 10^{-11} \; nt \; \mathrm{m^2/kg^2} \; (\text{or } \mathrm{m^3/kg\ sec^2}).$$

Implicit in the law of universal gravitation is the idea that the gravitational force between two particles is completely independent of the presence of other bodies or the properties of the intervening space.

Newton's Second Law of Motion states that the acceleration of a body is proportional to the force applied to it. Hence the familiar expression:

$$F = ma. \qquad (2)$$

Substituting the gravitational force for F, we have

$$a = \frac{F}{m} = G\frac{M_e}{R_e^2} = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.37 \times 10^6)^2}$$

$$= 9.81 \; \mathrm{m/sec^2} = g$$

where the Earth mass M_e and Earth radius R_e have been used to give the familiar g, acceleration due to the Earth's gravitational field at the Earth's surface. Thus it is seen that the acceleration of a body in a gravitational field is independent of the body's mass. (All bodies "fall" toward the Earth with the same acceleration.)

An expression for the variation in g as r varies may be obtained by differentiating expression (1) with respect to r.

$$dF = -2G\frac{Mm}{r^3}dr.$$

Dividing the above equation by equation (1):

$$\frac{dF}{F} = -2\frac{dr}{r}.$$

Substituting $a = g$ and differentiating expression (2) with respect to g we obtain:

$$dF = m\,dg$$

and dividing by expression (2) yields:

$$\frac{dF}{F} = \frac{dg}{g}.$$

Thus we see that

$$\frac{dg}{g} = -2\frac{dr}{r}$$

which shows that the fractional change in g is just twice that of r. For example, in going up 100 miles from the Earth's surface, r changes from about 4,000 miles to 4,100 miles, a relative change of 1/40. Therefore, g must change by about -1/20, or from about 9.80 m/sec^2 to about 9.31 m/sec^2. (This approximation is valid provided that dr/r is very small.)

302. REDUCTION OF THE TWO-BODY SYSTEM TO THE ONE-BODY SYSTEM

When two celestial bodies of comparable mass are considered to form a single, isolated system, that system is known as a two-body system. The motions of revolution of each individual body are about the center of mass of the system, which is located somewhere on the imaginary line in space between the two bodies, as shown in figure 302A.

Therefore, a description of the motion of body #1 with respect to body #2 (i.e., considering the center of the coordinate system to be at the center of body #2 rather than at the center of mass) gives rise to complicated mathematical expressions.

In the case of a satellite body of negligible mass compared to the main body, the center of mass of the system is extremely close to the center of the main body. When this situation exists, it is advantageous to assume that the center of mass of the system coincides with the center of the main body, creating a one-body problem rather than a two-body problem, thus simplifying the equations of motion of the satellite body with respect to the main body. The center of mass of the Earth-Moon system lies within the Earth, so the errors induced by the one-body system approximation for light, man-made satellites will be negligible indeed.

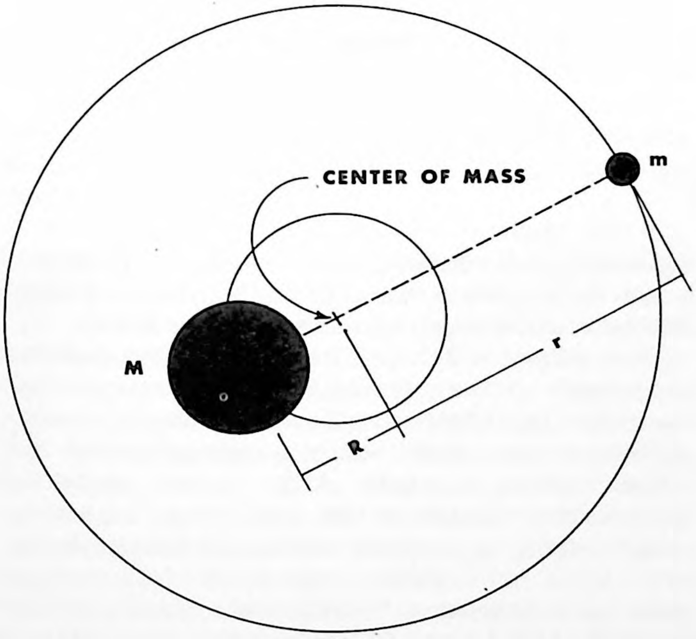

Figure 302A.–Rotation of a Two-Body System about the Center of Mass.

The problem is further simplified by the assumption of an ideal central force field, which implies that the gravitational force induced by the main body varies only with the radial distance from the center of the main body.

An example of the simplification which follows from the reduction to a one-body system is given below in the form of a determination of the period-distance relation for a satellite of negligible mass in orbit about the main body.

Considering figure 302A, the centrifugal force needed to keep each body on its circular path is equal to the gravitational force between the two bodies. The expression for the centrifugal force of circular motion is

$$F = \frac{mv^2}{r} = mr\omega^2,$$

where ω is the angular velocity. It follows from the definition of the center of mass that $mr = MR$, and, since the angular velocity must be equal for the two bodies, it can be said that $mr\omega^2 = MR\omega^2$. Therefore,

$$F = \frac{GMm}{(R+r)^2} = mr\omega^2 = MR\omega^2 .$$

Further, if we consider $r >> R$ (i.e., $M >> m$) then this expression reduces to

$$GM = \omega^2 r^3 .$$

The angular velocity ω may be expressed in terms of the period of one revolution T by the relation

$$\omega = \frac{2\pi}{T}$$

and the above expression becomes

$$GM = \frac{4\pi^2 r^3}{T^2}$$

or

$$T^2 = \frac{4\pi^2 r^3}{GM} . \qquad (3)$$

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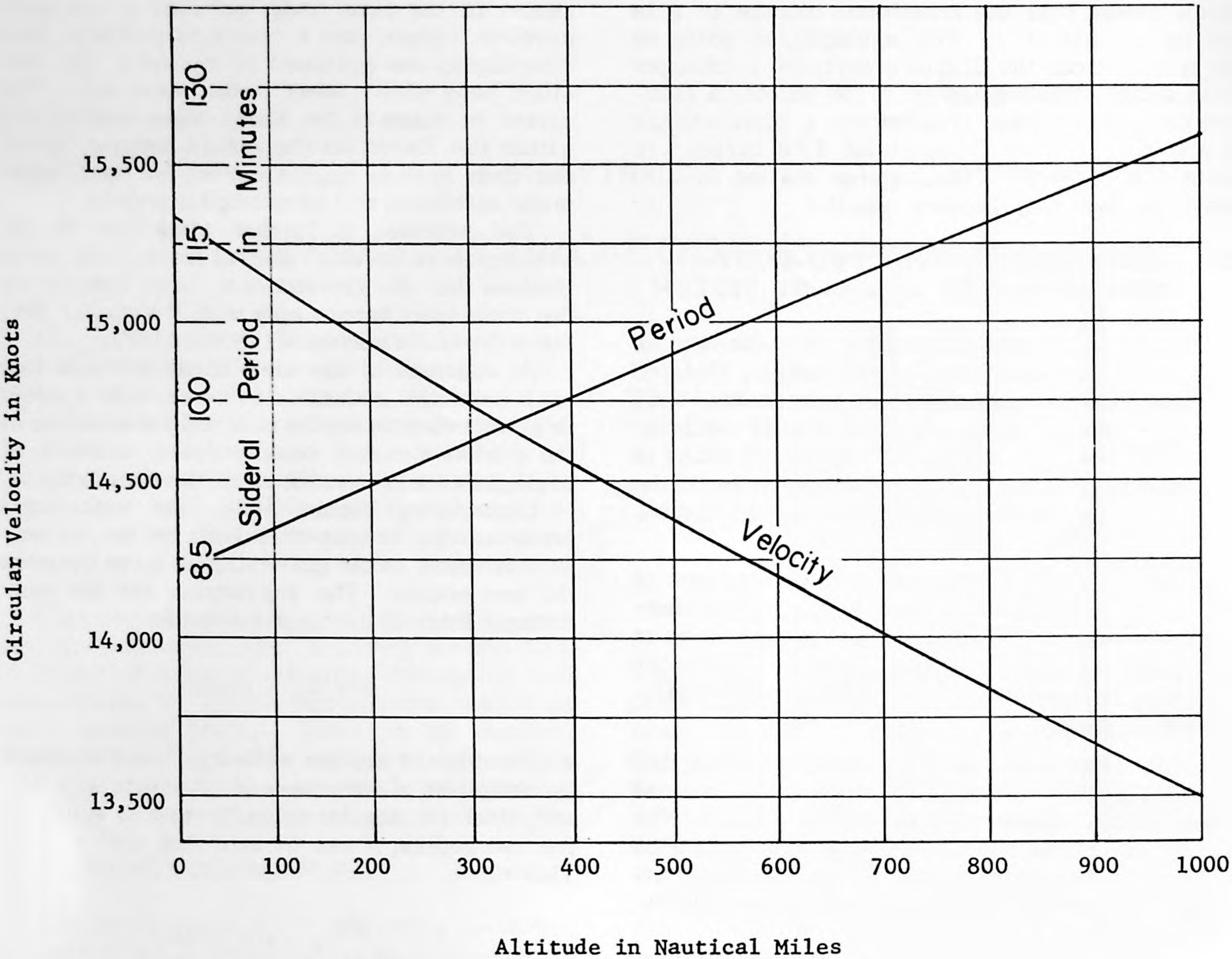

Figure 302B.–Variation of Circular Velocity and Sidereal Period with Altitude of Earth Satellites. For circular orbits, the velocity ratio is inversely proportional to the square root of the radii ratio:

$$\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}}.$$

Note that this simplified expression does not contain m, the mass of the orbiting body. Thus, theoretically, the mass of the Earth could be determined by observing the orbit radius and period of a satellite in a circular orbit about the Earth, without knowing the mass of the orbiting body. Figure 302B graphically illustrates equation (3).

303. DISCUSSION OF POSSIBLE ORBIT FORMS

Although a satellite body may orbit the central body along an infinite number of different paths, each path is a form of a well defined conic shape (assuming a central force field and a one-body system, i.e., the satellite is of negligible mass compared to the central body).

For a body which is "captured" by a central body, such as the planets in our solar system, the orbit is elliptical. For a "passing" body, such as a "one-shot" comet passing through the solar system, the path of the comet would be hyperbolic relative to the Sun. The remaining conic shapes, namely the circle and the parabola, are unique orbit paths. The circle is a special case of a "captured," elliptical orbit, and the parabola is the borderline case between elliptical and hyperbolic paths (see figure 303). The parabolic path is the idealized case where the

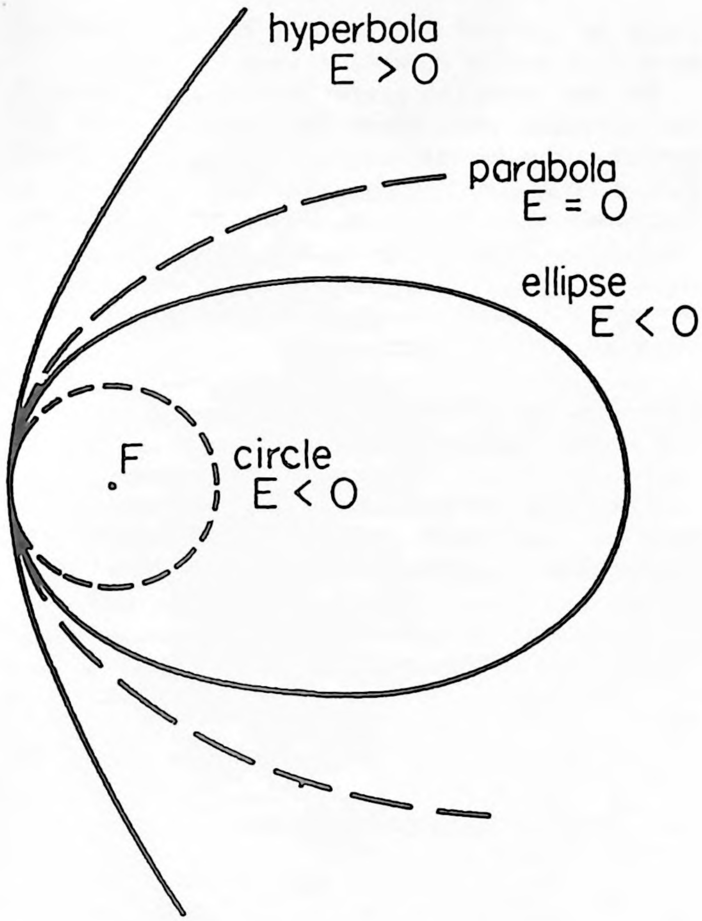

Figure 303.–Possible Orbit Forms.

body has zero energy with respect to the central body; no positive energy with which to escape, yet no negative energy to bind it in an orbit about the central body. (The energy relationships will be discussed further in the following sections.)

Needless to say, the circular and parabolic orbits are unstable, since any slight disturbance would cause the body to follow either an elliptical or hyperbolic path.

It is instructive, however, to study the behavior of bodies in circular orbits. Satellite bodies in circular orbits present a much simpler mathematical picture than when in generalized elliptical orbits, yet the basic energy and motion relationships are exactly similar in principle to the more generalized elliptical case. For example, the period-radius expression (3) developed for circular orbits becomes applicable to elliptical orbits merely by changing r, the constant radius of revolution, to a, the semimajor axis of the elliptical orbit.

Therefore, the next three sections and the applications which follow them deal with the special case of circular orbits, and the reader is reminded to bear in mind that the establishment of basic relationships is the intent here—discussion of the more general elliptical case to follow in sections 309 and 310.

This chapter discusses only the idealized orbit shapes, assuming perfectly spherical bodies, a central force field, and a one-body system. The actual shape of the Earth is, of course, not perfectly spherical but slightly pear shaped. This asymmetry causes "perturbations" of the orbits of close-in Earth satellites which are discussed in section 806.

304. POTENTIAL ENERGY OF THE CIRCULAR ORBIT

The potential energy of a very small body at a distance r from a very large body may be defined as the work required to bring the small body from infinity to a distance r from the large body.

The work required may be found by integrating the force expression with respect to r from ∞ to r.

$$P.E. = \int_{\infty}^{r} F\,dr = \int_{\infty}^{r} \frac{GMm}{r^2}\,dr = -\left.\frac{GMm}{r}\right|_{\infty}^{r}$$

$$P.E. = -\frac{GMm}{r}.$$

The result indicates that the potential energy of a body in orbit is always negative. This corresponds to the fact that the force is attractive between the bodies, e.g., to separate two bodies with -500 joules potential energy (attractive), +500 joules of work would have to be done on the bodies. Then the net potential energy of the

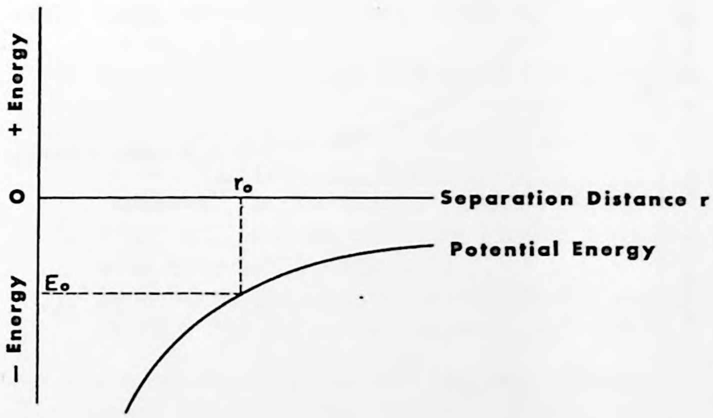

Figure 304.–Potential Energy vs. separation distance. The Graph shows how much energy must be supplied to completely separate (i.e., to ∞) two bodies which are a distance r.

system would be zero and the bodies would no longer attract one another.

305. KINETIC ENERGY OF THE CIRCULAR ORBIT

The expression for kinetic energy of a moving body is

$$K.E. = \frac{1}{2} mv^2$$

and, for circular motion, $v = \omega r$ may be substituted to obtain

$$K.E. = \frac{1}{2} m\omega^2 r^2.$$

From an equation preceding equation (3), we have $GM = \omega^2 r^2$, or $\omega^2 r^2 = GM/r$ for circular orbits. Therefore we have:

$$K.E. = \frac{GMm}{2r} \qquad (5)$$

for a body m in a circular orbit at radius r from the central body M. Note that this is similar in form to the potential energy expression, but that the $K.E.$ is always positive. This corresponds to the ability of the moving body to do work in bringing its energy to zero. Whereas, in the previously discussed potential energy case, work had to be applied to separate the bodies and bring their energy to zero. For example, a 1 kg hammerhead moving at 2 m/sec could do $1/2\ mv^2 = 1/2 \times 1 \times (2)^2 = 2$ joules of work on a nail in coming to rest.

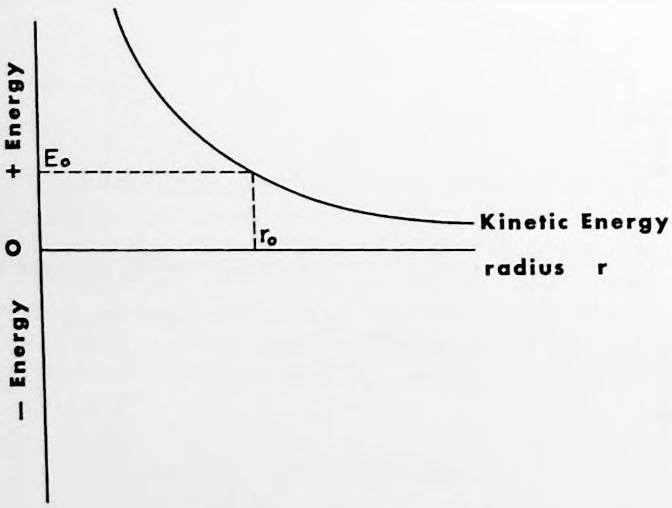

Figure 305.–Kinetic Energy vs. Radius r for a body m in a circular orbit about a central body M. Note that there is a particular and unique kinetic energy for each radius, and vice-versa.

We see from the graph that as the radius of the circular path about the central body decreases, the kinetic energy increases. From $K.E. = (1/2)m\omega^2 r^2$, it is apparent that if $K.E.$ increases as r decreases, then ω must increase. Thus, the closer to the central body, the faster must the orbiting body rotate. And, as the radius of rotation becomes very large, the angular velocity becomes very small.

306. TOTAL ENERGY OF THE CIRCULAR ORBIT

The total energy of a body is the sum of its potential and kinetic energies. So, combining expressions (4) and (5), we have, for a circular orbit:

$$T.E. = K.E. + P.E.$$

$$= \frac{GMm}{2r} - \frac{GMm}{r} \qquad (6)$$

$$T.E. = -\frac{GMm}{2r}.$$

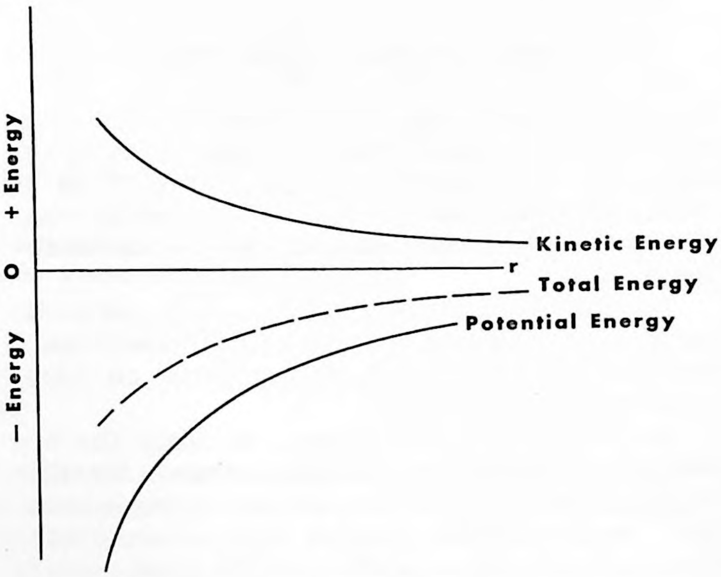

Figure 306.–Total Energy in a Circular Orbit.

We see from the expression for $T.E.$ and from figure 306, which show the sum of $K.E.$ and $P.E.$, that the total energy is negative for a body in orbit. This is to be expected, for it indicates that the body is "captured" in the system, i.e., positive work must be done to free the body from the gravitational force.

307. APPLICATIONS

Application #1:

Assume that a space probe of mass 100 kg was shot from the Earth with an initial velocity of 12 km/sec. It is desired to know whether the probe will orbit the Earth (assume a circular orbit), or whether it will escape the Earth's gravitational field. Neglect any atmospheric effects.

The answer can be obtained by determining the total energy of the probe: if the total energy is negative, the probe is "captured," and it will go into orbit; if the total energy is positive, then the probe will escape from the Earth's attraction. However, equation (6) for total energy cannot be used, since that expression applies only to bodies already in orbit. Using the law of conservation of energy we can say that the total energy of the probe is equal to the sum of the $K.E.$ and $P.E.$ at takeoff.

$$T.E. = (K.E. + P.E.) \text{ in space}$$

$$= (K.E. + P.E.) \text{ at takeoff}$$

$$P.E. \text{ at takeoff} = -\frac{GM_e\, m_p}{r_e}$$

$$= -\frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})(100)}{(6.37 \times 10^{6})}$$

$$= -\ 6.25 \times 10^{9} \text{ joules}$$

$$K.E. \text{ at takeoff} = \frac{1}{2} m_p\, v^2$$

$$= \frac{1}{2}(100)\,(12 \times 10^3)^2$$

$$= 7.20 \times 10^{9} \text{ joules}$$

$$T.E. = P.E. + K.E.$$

$$= -\ 6.25 \times 10^{9} + 7.20 \times 10^{9}$$

$$\underline{T.E. = +\ 9.5 \times 10^{8} \text{ joules.}}$$

Thus, since the total energy relative to the Earth is positive, the probe will not become a satellite of the Earth.

Application #2:

As a continuation to the above problem, what would be the smallest takeoff velocity which would enable a space vehicle to escape from the Earth? Neglect any atmospheric effects.

This would be the case when the $P.E.$ and $K.E.$ at takeoff were exactly equal so that the $T.E.$ would be zero.

$$T.E. = 0 = P.E. + K.E.$$

or

$$\frac{1}{2} m_p\, v^2 = \frac{GM_e\, m_p}{r_e}$$

$$v^2 = \frac{2\,GM_e}{r_e}\,.$$

So

$$\underline{v = 11.2 \text{ km/sec}} = 6.96 \text{ mi/sec}$$

$$= 25{,}000 \text{ mi/hr} = 21{,}750 \text{ kt.}$$

Any initial velocity less than this would place the body in orbit, and any velocity greater than this would allow the body to escape from the Earth's attraction. (Initial velocity here means, of course, the maximum velocity — reached at the end of the power period.) Again we assume that there are no losses to the atmosphere, etc., during takeoff.

Application #3:

Assume a 100 kg satellite (rocket #1), has been in a circular orbit and the period has been noted to be exactly 3 hours. It is desired to place another 100 kg satellite (rocket #2) in orbit along side the original satellite.

a. What will be the radius of the orbit of rocket #2?
b. What should be the "escape" velocity of rocket #2 from the Earth?
c. What will be its linear velocity in orbit?
d. What will be its relative linear velocity with respect to rocket #1?
e. How would the results be affected if rocket #2 was 200 kg mass rather than 100 kg?

a. From equation (3) we see that the radius and the period are directly related. The M, which corresponds to the central body, would in this case be the mass of the Earth. Orbit radius will be indicated by r_o; Earth radius by r_e. The radius of the orbit of rocket #1 is therefore:

$$r_o^3 = \frac{G M T^2}{4\pi^2}$$

$$= \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})\,(3)^2(3600)^2}{4\pi^2}$$

$$= 1.176 \times 10^{21} \text{ m}^3$$

So

$$\underline{r_o = 10.56 \times 10^3 \text{ km} = 5{,}700 \text{ N. miles.}}$$

Since it was desired to place the second rocket in orbit adjacent to the first, this then is the desired radius of the #2 rocket orbit. (Since the radius of the Earth is about 3,444 N. miles, the rocket will be about 2,256 N. miles above the surface.)

b. To find the necessary "escape" velocity, the energy concept can be used. The total energy in orbit can be found from equation (6) and the potential energy on the Earth's surface can be found from equation (4). Since the total energy of the system remains constant (assuming that there are no losses and that no power is applied after blastoff) we can equate the total energy to the sum of kinetic and potential energy at any common point.

$$T.E.\ (\text{in orbit}) = (P.E. + K.E.)\ \text{on Earth}$$

or

$$K.E.\ (\text{on Earth}) = T.E.\ (\text{in orbit}) - P.E.\ (\text{on Earth})$$

$$\frac{1}{2} m_p v^2 = -\frac{1}{2} G \frac{M_e m_p}{r_o} - \left(-G \frac{M_e m_p}{r_e}\right)$$

$$v^2 = -G \frac{M_e}{r_o} + 2G \frac{M_e}{r_e}$$

$$v^2 = G M_e \left(\frac{2}{r_e} - \frac{1}{r_o}\right) \qquad (7)$$

$$= (6.67 \times 10^{-11})(5.97 \times 10^{24})$$

$$\left(\frac{2}{6.37 \times 10^6} - \frac{1}{10.56 \times 10^6}\right)$$

$$= 8.73 \times 10^7 \text{ m}^2$$

So

$$\underline{v_e = 9.35 \text{ km/sec}}$$

$$= 18{,}100 \text{ kt "escape" velocity.}$$

c. and d. It can be seen from equation (3) that the orbit radius and period are uniquely related. Therefore, since the radii of the two rockets are to be equal, the periods must also be equal, namely 3 hours. This naturally makes the angular velocities of the two, as well as the linear velocities, equal.

$$v = \omega r.$$

Making use of equation (3):

$$v^3 = \omega^3 r^3 = \frac{(2\pi)^3}{T} \quad \frac{(G M_e T^2)}{4\pi^2} = \frac{2\pi G M_e}{T}$$

$$= \frac{2\pi\ (6.67 \times 10^{-11})(5.97 \times 10^{24})}{3\ (3600)}$$

$$= 232 \times 10^9 \text{ m}^3/\text{sec}^3$$

So

$$\underline{v_o = 6.15 \text{ km/sec} = 13{,}750 \text{ mi/hr}}$$

$$= 11{,}930 \text{ kt orbit velocity.}$$

Since the two satellites have the same radius and therefore the same period, the relative velocity is forced to be zero. It must be kept in mind that a given radius specifies a specific and unique velocity, so that two bodies can never be in the same circular orbit at different speeds. The orbit velocity will be constant only in the case of circular orbits.

e. This question is included to emphasize that the mass of the orbiting body, m_p, does not appear in the expression for escape velocity from the Earth, or in the expression connecting period and radius. Thus the escape velocity required for the 200 kg probe is exactly the same as for the 100 kg probe. Naturally, the energy required to attain the escape velocity is greater for the 200 kg probe, but the velocity required to obtain a certain orbit is independent of mass. Once in orbit, we can see from equation (3) that the period is also independent of mass, depending only on the radius. NOTE: Chapter 10, sections 1004, 1005, and 1006 show that placing a satellite in a circular orbit alongside a satellite previously in orbit would be a formidable task indeed! This problem is included here to emphasize the basic relationships involved — not the shortsightedness of the author.

Application #4:

Figure 307.–A Satellite in Orbit.

Suppose that a satellite in orbit about the Earth fires rockets in a direction which is tangent to the orbit. It is noted some time later that the period of the satellite has increased.

Were the rockets fired aft (booster rockets), or were they fired forward (retrorockets)?

We see from equation (3):

$$T = \frac{4\pi^2 r_o^3}{GM_e},$$

that if the period T has increased, then the radius of the orbit has also increased. From the total energy expression (6):

$$T.E. = -\frac{GM_e m_s}{2r_o},$$

and from figure 306, we see that if r_o increases then $T.E.$ becomes a smaller negative number, i.e., the $T.E.$ has become more positive. Thus, the total energy of the system has increased (become more positive) by the firing of the rockets. This was accomplished by attempting to increase the $K.E.$ by firing booster rockets aft. (It will be seen in chapter 10 that a single firing of rockets to change the orbit will result in a change in shape of the orbit from circular to elliptical.)

To apply power in the direction of motion by firing booster rockets with the net result of reducing the average velocity is perhaps not intuitively obvious at first inspection. We must realize, however, that the rocket has moved to a larger orbit, reducing the hold of potential energy on the rocket. As more power was applied in the direction of motion (adding energy), the rocket would move into larger and larger orbits at decreasing velocities, finally coming to a complete stop at an infinite distance from the Earth with $T.E. = 0$, were there no other bodies in the universe to interfere.

As an exercise, analyze the situation where the retrorockets are fired.

308. DISCUSSION OF CONICS

The previous section of this chapter included a discussion of the circular orbit which, as was previously pointed out, is a special case of the more general and more common elliptical orbit. Actually, a circular orbit is often desired, since it has many practical advantages for Earth-satellite applications, and it is often a good approximation to the behavior of an elliptical orbit with small eccentricity (i.e., nearly circular). Also, the general energy principles which have been stressed hold true for elliptical orbits as well, and other relations will be seen to be similar in form. For example, the velocity of a body in any conic orbit at a distance r from a central body M is given by

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) \tag{8}$$

where a is the semimajor axis of the conic orbit. (Note the similarity of expression (8) to expression (7) which was given in application #3b, section 307.)

At this point, perhaps it would be appropriate to include a review of conic curves in general and elliptical curves in particular.

In general form, a conic is defined to be the locus of a point P relative to a fixed point 0 and a "directix" line, which is given by the form

$$\frac{R}{d - R\cos\theta} = e = \text{constant} \tag{9a}$$

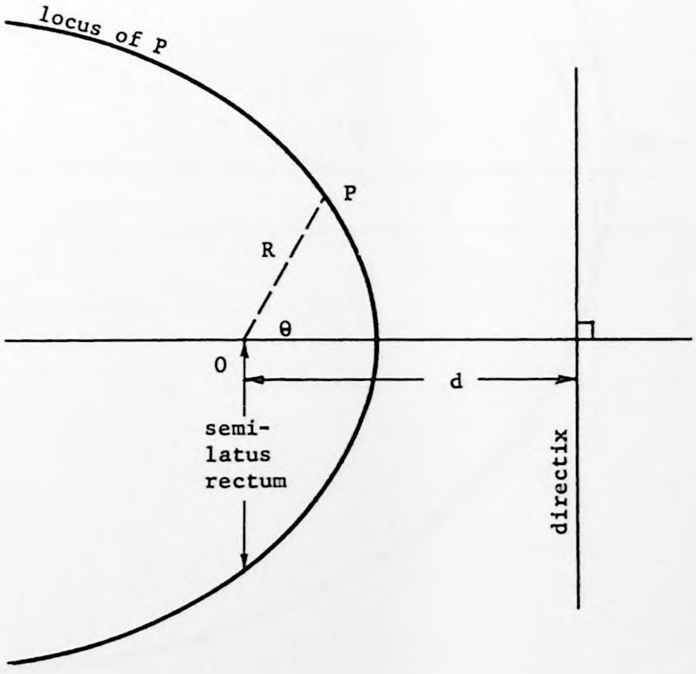

Figure 308A.–The Definition of a Conic.

where the constant e is known as the "eccentricity" of the curve and is an indication of the shape of the curve, a low eccentricity indicating a near-circular shape, and a high eccentricity indicating a long narrow shape. For the various curves:

1. circle: $e = 0$
2. ellipse: $0 < e < 1$
3. parabola: $e = 1$
4. hyperbola: $e > 1$
5. straight line: $e \to \infty$.

Expression (9a) is often expressed in a different form for convenience. For example, in polar coordinates, where one of the foci is located at the origin, the general conic form is

$$r = \frac{p}{1 + e \cos \theta}$$

where p is the semilatus rectum; or

$$\frac{1}{r} = C\,(1 + e \cos \theta)$$

where

$$C = \frac{1}{de}\,.$$

The Ellipse

The conic known as the ellipse is the locus of a point such that the sum of its distances from two fixed points is constant. (In figure 308B, $F'PF = 2a$, the length of the major axis.) The equation for the ellipse, given now in

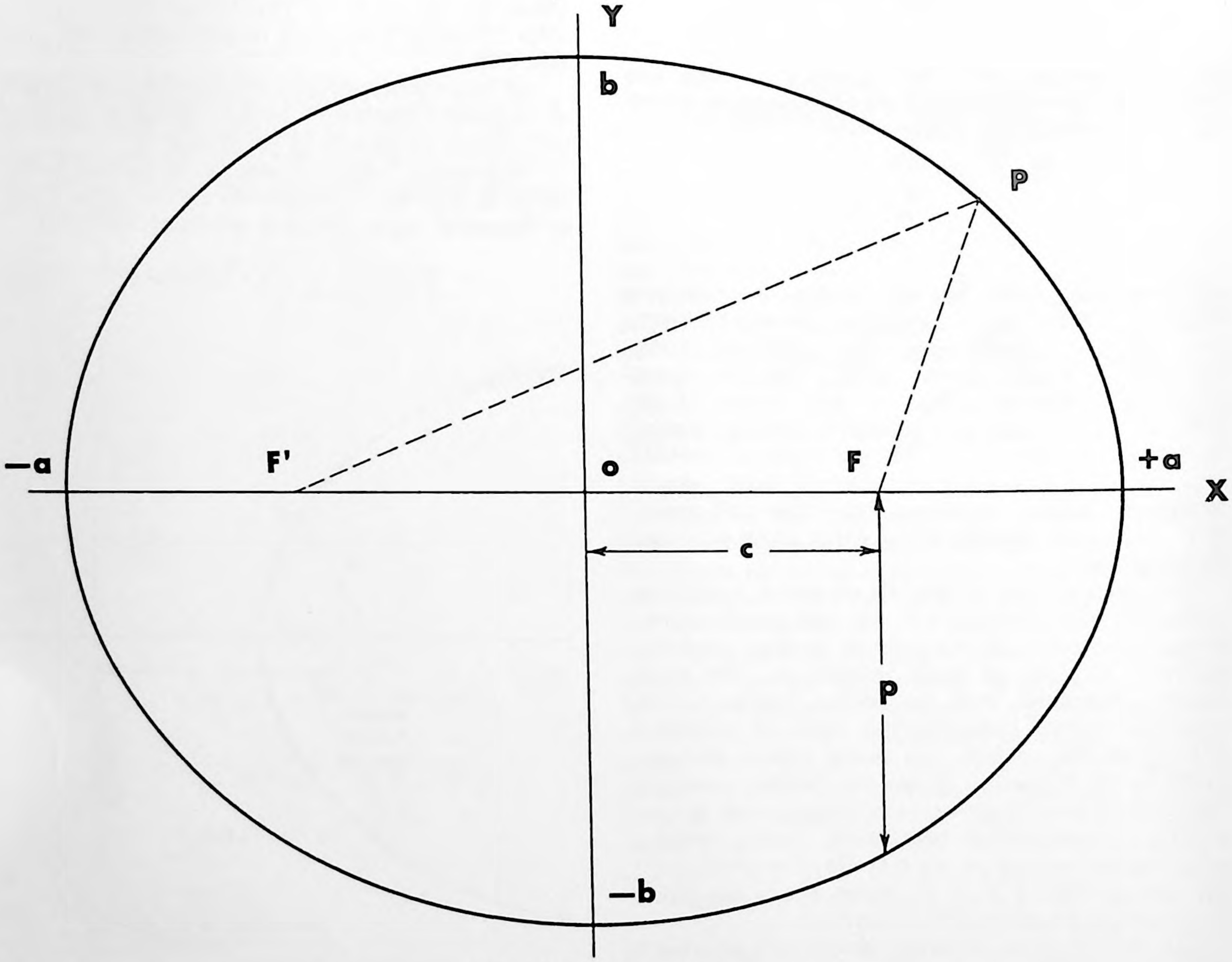

Figure 308B.–The Ellipse.

rectangular coordinates, follows directly from the definition:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (10a)$$

where

$$b^2 = a^2 - c^2 . \qquad (10b)$$

In polar coordinates, the equation for the ellipse would be in the form of (9b) or (9c), where the eccentricity e would be positive but less than 1. Equations (9b) and (9c) assume the origin to be at one focus; equation (10a) assumes the origin to be at the center of the ellipse.

The length of the semilatus rectum p is determined by substituting $x = c$ into equation (10a) and solving for y:

$$p = \frac{b^2}{a} . \qquad (10c)$$

The eccentricity for an ellipse is the ratio of c to a:

$$e = \frac{c}{a} . \qquad (10d)$$

Thus, if the foci have a larger separation (2c) relative to the major axis (2a), then the eccentricity, or deviation from circular, is greater, as can be quickly seen from figure 308B. In the above figure, $e = 0.54$.

By manipulating expressions (10b) and (10d), several forms can be obtained for the eccentricity e, semimajor axis a, and semiminor axis b. For example:

$$b = a\sqrt{1 - e^2} . \qquad (10e)$$

And expressions for e, a, and b using the terms of equations (9a), (9b), or (9c) can be obtained. For example, the semimajor axis a is seen to be equal to one-half the sum of the maximum radius and the minimum radius.

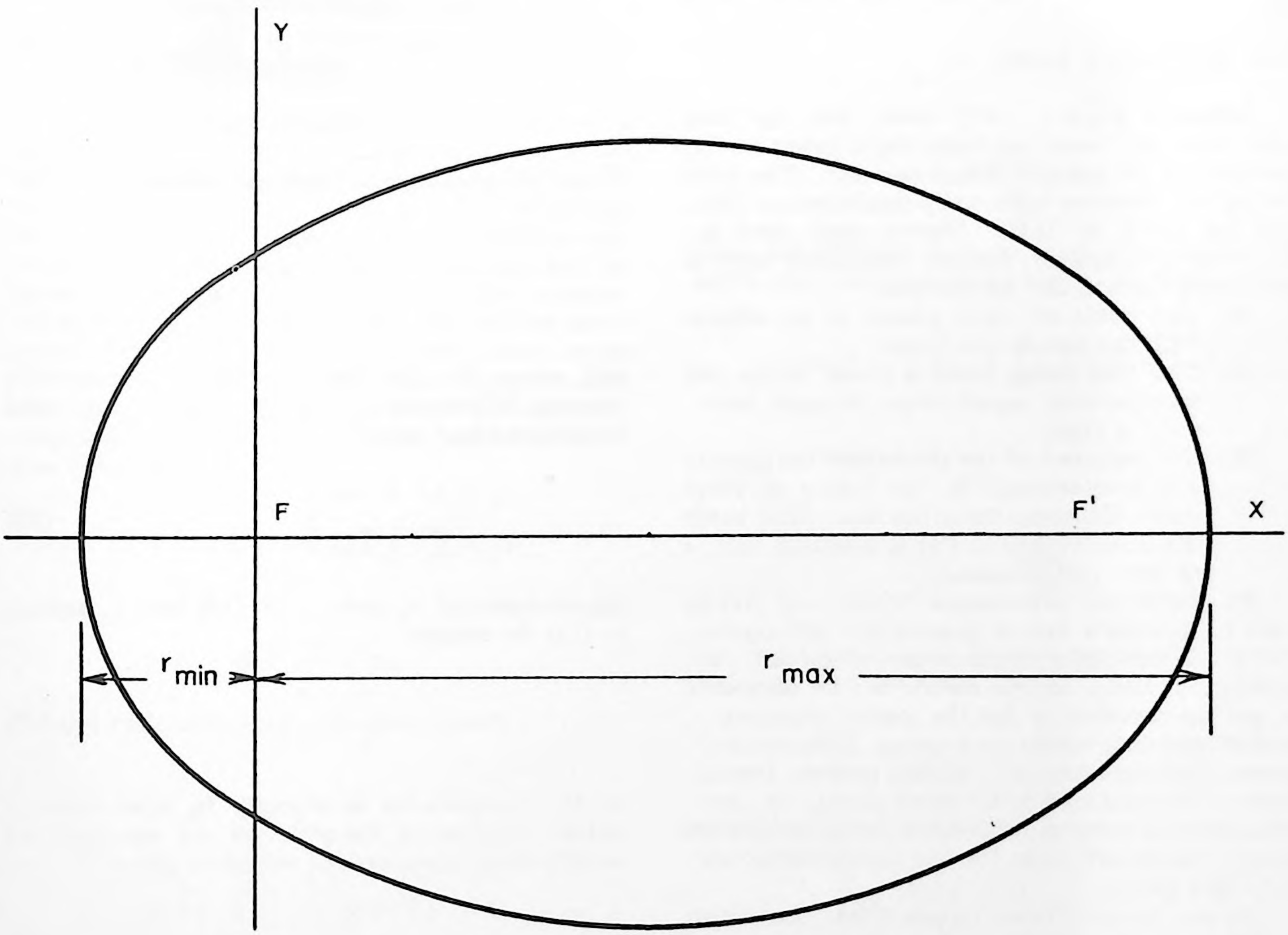

Figure 308C.–Maximum and Minimum Radii in the orbit ellipse.

Using the form of (9c), where

$$\cos\theta = -1 \text{ for } r_{min},$$

$$\cos\theta = +1 \text{ for } r_{max},$$

this relation may be expressed as:

$$a = \frac{1}{2}\left(r_{min} + r_{max}\right) = \frac{1}{2}\left(\frac{1}{C(1-e)} + \frac{1}{C(1+e)}\right)$$
$$a = \frac{1}{C(1-e^2)}. \tag{10f}$$

And, using (10e), we have

$$b = a\sqrt{1-e^2} = \frac{1}{C\sqrt{1-e^2}}. \tag{10g}$$

Or, using (10f),

$$b = \sqrt{a}\,\sqrt{a}\,\sqrt{1-e^2} = \sqrt{\frac{a}{C}}. \tag{10h}$$

309. KEPLER'S LAWS

Johannes Kepler (1571-1630) was the man who first laid down the laws which describe the motions of the planets about the Sun. The first two of the famous laws were published in 1609, and the third in 1619. Newton then used the three laws of Kepler, and his own three laws of planetary motion are as follows:

I. The orbit of each planet is an ellipse with the Sun at one focus.
II. The line which joins a planet to the Sun sweeps over equal areas in equal intervals of time.
III. The squares of the periods of the planets are proportional to the cubes of their mean distances from the Sun. (The mean distance refers to the semimajor axis a of the orbit ellipse.)

To obtain an expression which will verify Law I, Newton's law of gravitation and expressions for the force components along and perpendicular to the radius vector can be combined to get an expression for the radial distance r, which is recognizable as a conic. (For our purposes, the motions of a planet may be considered to be confined to its orbit plane, i.e., two-dimensional motion.) So, let us first review the polar coordinate form for the acceleration vectors in a plane.

As can be seen from figure 309A, the vector U in the XY plane can be expressed as components along and perpendicular to the radius vector by

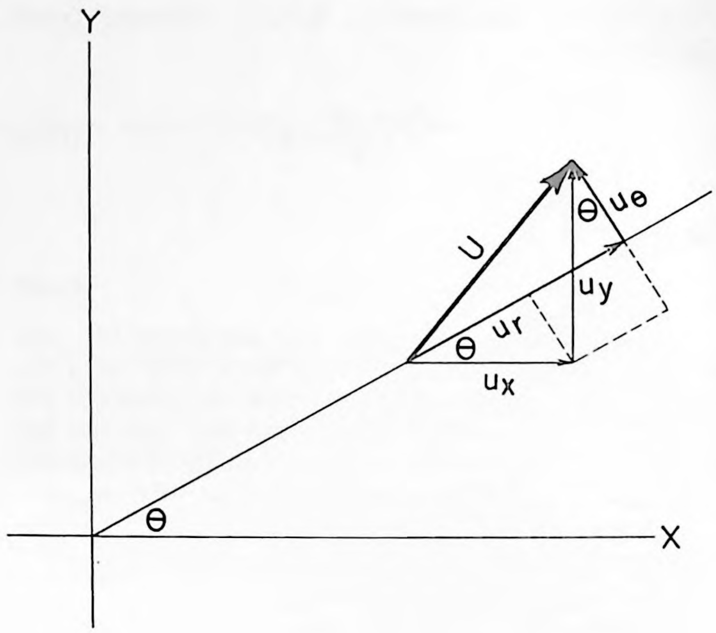

Figure 309A.—The Relation Between Rectangular and Polar Components of a Vector.

$$u_r = u_x \cos\theta + u_y \sin\theta$$
$$u_\theta = -u_x \sin\theta + u_y \cos\theta. \tag{11}$$

The relation between polar and rectangular coordinates is

$$x = r\cos\theta$$

$$y = r\sin\theta$$

and, using dots to denote differentiation with respect to time, the velocity components would therefore be given by

$$v_x = \dot{x} = \dot{r}\cos\theta - r\dot{\theta}\sin\theta$$
$$v_y = \dot{y} = \dot{r}\sin\theta + r\dot{\theta}\cos\theta. \tag{12}$$

On substituting v_x and v_y of (12) for u_x and u_y of (11) we obtain:

$$v_r = \dot{r}$$
$$v_\theta = r\dot{\theta} \tag{13}$$

as the components of velocity in polar coordinates. Continuing the process, we see that the acceleration components would be given by

$$a_x = \ddot{x} = (\ddot{r} - r\dot{\theta}^2)\cos\theta - (r\ddot{\theta} + 2\dot{r}\dot{\theta})\sin\theta$$
$$a_y = \ddot{y} = (\ddot{r} - r\dot{\theta}^2)\sin\theta + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\cos\theta \tag{14}$$

and substitution of a_x and a_y of (14) for u_x and u_y of (11) yields

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} - \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt} \tag{15}$$

as the components of acceleration in polar coordinates. The polar coordinate form for both the conic curve and the acceleration have been given because, when dealing with orbit problems, the polar form lends itself to easier manipulation and recognition of conic forms. With these expressions at hand, we are prepared to derive Kepler's three laws of planetary motion.

Derivations

The following derivations, developed for planetary bodies, assume passive satellites in a closed system in which no energy is added or removed.

Law I: The orbit of each planet is an ellipse with the Sun at one focus.

The basic relationship to be used here is merely $F = ma$. Two components of the force will be considered; the component along the radius vector, and the component perpendicular to the radius vector. These two components may be used to produce two equations, for the force along the radius vector is the gravitational force, given by equation (1), and the force perpendicular to the radius vector is zero (since the force of gravity acts only along the line joining two bodies). Expressions for the corresponding acceleration are obtained from equation (15). So, we have

$$F_r = -\frac{GmM}{r^2} = m\ddot{r} - mr\dot{\theta}^2$$

and (16)

$$F_\theta = 0 = mr\ddot{\theta} + 2m\dot{r}\dot{\theta}\,.$$

Let the following terms be introduced:

a. $u = \dfrac{1}{r}$

b. $h = r^2\dot{\theta}$

c. $\dot{r} = \dfrac{dr}{d\theta}\dfrac{d\theta}{dt} = \dfrac{dr}{d\theta}\dot{\theta}\,.$

NOTE: Substitution a. is often used for convenience in the solution of differential equations. Since angular momentum is equal to $mr^2\dot{\theta}$, h in substitution b. above is the angular momentum per unit mass: the specific angular momentum. Then, from c. and b.,

$$\dot{r} = \frac{dr}{d\theta}\frac{h}{r^2}$$

and

$$\frac{du}{d\theta} = \frac{d}{d\theta}\left(\frac{1}{r}\right) = -\frac{1}{r^2}\frac{dr}{d\theta}$$

so

$$\dot{r} = -r^2\frac{du}{d\theta}\frac{h}{r^2} = -h\frac{du}{d\theta}\,.$$

Differentiating this expression yields

$$\frac{d^2r}{dt^2} = \ddot{r} = -h\frac{d}{dt}\left(\frac{du}{d\theta}\right) = -h\frac{\partial}{\partial\theta}\left(\frac{du}{d\theta}\right)\dot{\theta}$$
$$= -h\frac{d^2u}{d\theta^2}\dot{\theta} = -h\frac{d^2u}{d\theta^2}hu^2$$
$$\ddot{r} = -h^2u^2\frac{d^2u}{d\theta^2}\,.$$

Now, substituting $\ddot{r}$ from above and $\dot{\theta}$ from b. above into the expression for F_r of (16) we have

$$-GmMu^2 = -mh^2u^2\frac{d^2u}{d\theta^2} - m\frac{1}{u}(h^2u^4)$$
$$-GM = -h^2\frac{d^2u}{d\theta^2} - h^2u.$$

Or

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2}\,.$$

The solution of this second-order differential equation is of the form

$$u = \frac{GM}{h^2} + A\cos\theta$$ (A is a constant of integration)

which can be written in the form

$$\frac{1}{r} = \frac{GM}{h^2}\left(1 + \frac{Ah^2}{GM}\cos\theta\right)$$
$$= C(1 + e\cos\theta)$$

or in the form

$$r = \frac{\frac{h^2}{GM}}{1 + \frac{h^2 A}{GM} \cos \theta}$$

$$= \frac{p}{1 + e \cos \theta} .$$

This is recognized as a conic in the form of equation (9c) or (9b) which indicates that the orbit must be in the form of an ellipse. Thus, Kepler's first law has been demonstrated.

It should be pointed out that the solution of Kepler's first law implies four constants of integration, since the final form is obtained from differential equations which are second order in both the r and θ parameters. We began by assuming that the motion was confined to the orbit plane, thus eliminating the ϕ coordinate which would have also had derivatives of the second order. Thus six constants of integration are actually obtained in the determination of the position expression of a body rotating in three-dimensional space. Had the problem been solved using the rectangular coordinate system, second derivatives of x, y, and z would have appeared in the solution, again implying six constants of integration. These six parameters

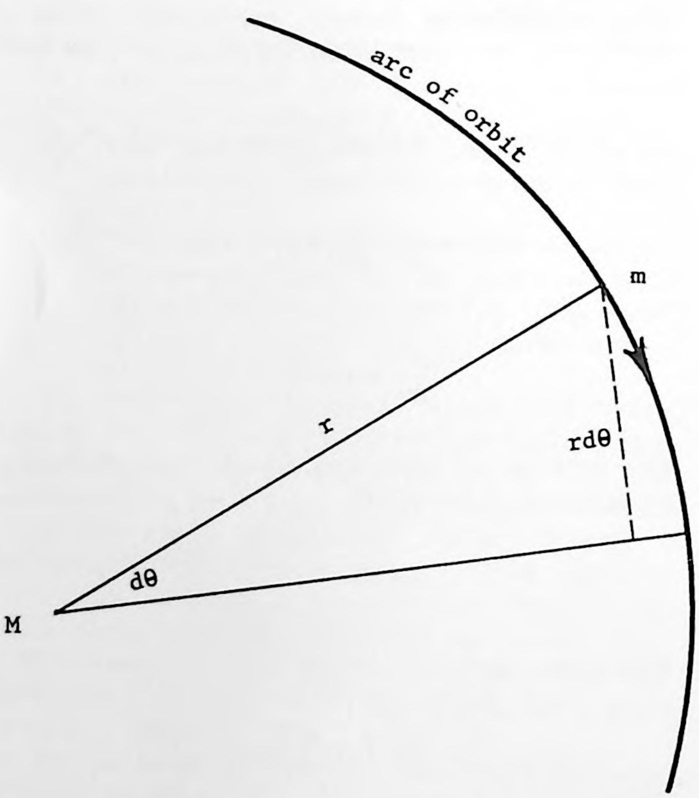

Figure 309B.–The Differential Area of an Arc.

are called "orbital elements." They describe the position of the body in space by giving the orientation of the orbit plane, the size and shape of the orbit, and the velocity and position of the body for a specific time, thus determining the location of the body in three-dimensional space for <u>all</u> time. The orbital elements and the various forms which they may take will be discussed more completely in the first sections of the next chapter.

<u>Law II: The line which joins a planet to the Sun sweeps over equal areas in equal intervals of time.</u>

Multiplying equation ($16\text{-}F_\theta$) by r/m gives

$$O = 2 r \dot{r} \dot{\theta} + r^2 \ddot{\theta}$$

$$O = \frac{d}{dt} (r^2 \dot{\theta})$$

from which we see that $r^2 \dot{\theta} = h$ is a constant. As can be seen from figure 309B, the differential area of an arc is given by $A = (1/2) r^2 d\theta$. Therefore the area swept per unit time is given by

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \dot{\theta}.$$

But we have just determined that $r^2\dot{\theta}$ was a constant. Therefore, the area per unit time is constant, which is Kepler's second law. Angular momentum is given by $r m v = m r^2 \omega$. Since $r^2 \dot{\theta} = r^2 \omega$, we see that <u>Kepler's second law is also a statement that the angular momentum is constant.</u>

<u>Law III: The squares of the periods of the planets are proportional to the cubes of their mean distances from the Sun.</u>

It was shown in the previous discussion that

$$\frac{dA}{dt} = \frac{1}{2} r^2 \omega .$$

The area A is found by integrating over one complete sweep, which is one period of time T.

$$A = \int_0^T \frac{dA}{dt} dt = \int_0^T \frac{\omega r^2}{2} dt = \frac{1}{2} \omega r^2 T .$$

The area for an ellipse is πab. So, from above,

$$T = \frac{2\pi\, a\, b}{r^2}\ .$$

Let us use the form for b given as equation (10h):

$$b = \sqrt{\frac{a}{C}},$$

where C was seen to be

$$\frac{GM}{h^2} = \frac{GM}{(r^2\,\omega)^2}$$

in the derivation of Law I. So, we have

$$b = \frac{\sqrt{a}}{\sqrt{GM}}\, r^2\, \omega\ .$$

Substituting this form for b in the above expression for T yields

$$T = \frac{2\pi\, a\, \sqrt{a}\ \ r^2\,\omega}{\sqrt{GM}\ \ r^2\,\omega} = \frac{2\pi\, a^{3/2}}{\sqrt{GM}}\ .$$

Or

$$T^2 = \frac{4\pi^2\, a^3}{GM}$$

which is Kepler's third law. A similar form was given as equation (3). So, for bodies at different distances from the central force,

$$\frac{(T_1)^2}{(T_2)^2} = \frac{(a_1)^3}{(a_2)^3}\ .$$

Naturally, there are several methods of proving Kepler's laws. Other proofs may be found in the references cited at the end of this chapter.

It should be pointed out at this time that Kepler's laws assume that the planets have a negligible mass compared to the Sun. Saying it another way, the center of rotation, or the center of gravity of the system, is taken to be at the center of the Sun. In the case of a planet such as Jupiter, whose mass is about 1% that of the Sun's mass, Kepler's laws do not strictly hold, but give only a good first approximation to the actual motion. (Note section 302 where the assumption that $M >> m$ was used to arrive at a form of Kepler's third law.)

Also, when a central force field is assumed, this means that the force is to be a function only of the radial distance r from the center of the central body. However, in the case of close-in orbits of satellites, the Earth's "pear" shape causes irregularities in the gravitational field which induce (1) rotation of the orbit plane about the north-south Earth pole, and (2) rotation of the axis of the orbit ellipse in the orbit plane (called rotation of perigee). (See section 806.)

So for a more exact analysis it is seen that Kepler's laws are an idealized case which becomes the first approximation, but which does not completely describe the motion of a rotating body.

310. ENERGY EXPRESSION DERIVATION

In the first sections of this chapter, energy relations were discussed and used in illustrative examples. The equations of motion (16) will now be used to obtain these energy expressions. Multiplying the first equation by $\dot{r}$ and the second equation by $r\,\dot{\theta}$ we have

$$-\frac{GMm}{r^2}\,\dot{r} = m\,\ddot{r}\,\dot{r} - m\,r\,\dot{r}\,(\dot{\theta})^2$$

$$0 = 2m\,r\,\dot{r}\,(\dot{\theta})^2 + m\,r^2\,\dot{\theta}\,\ddot{\theta}\ .$$

Adding these equations yields

$$0 = +\frac{GMm}{r^2}\,\dot{r} + m\,r\,\dot{r}\,(\dot{\theta})^2 + m\,\ddot{r}\,\dot{r} + m\,r^2\,\dot{\theta}\,\ddot{\theta}.$$

To obtain the energy, the above equation is integrated with respect to time. (The area under a force-time curve is the energy.) The result of the integration is

$$E = -\frac{GMm}{r} + \frac{1}{2}\,m\,(\dot{r})^2 + \frac{1}{2}\,m\,r^2\,(\dot{\theta})^2$$

$$E = -\frac{GMm}{r} + \frac{1}{2}\,m\,(\dot{r}^2 + r^2\,\dot{\theta}^2) \qquad (17)$$

where E is a constant and is the total energy of the system. Note that the second factor of (17) contains the squares of the velocity components in polar coordinates. So (17) could be written in the form

$$E = -\frac{GMm}{r} + \frac{1}{2}\,m\,v^2 \qquad (17)'$$

which is recognized as the expression for total energy previously used. Also, if the total energy E is negative, then the radius r cannot go to infinity, for this would cause the first term of (17) to go to zero, leaving only the second, positive term. Thus, for a bounded conic orbit (an ellipse), the total energy is negative, and conversely a negative total energy means an

elliptical orbit, as was previously pointed out in section 306.

Equation (17) holds for all points of the orbit, i.e., all values of r in the orbit. At perigee $\dot{r}$ vanishes since the curve is perpendicular to the radius vector at that point. Also, at perigee, $r = a(1 - e)$ for the ellipse. We can use these perigee conditions in (17) to solve for E. Substitute $\dot{\theta}^2 = h^2/r^4$ where, as can be seen from the second form obtained in the derivation of Law I,

$$h^2 = GMp = GM\,\frac{b^2}{a} = GMa(1 - e^2).$$

Solving for E,

$$E = \frac{m}{2}\;\frac{GMa(1 - e^2)}{a^2(1 - e)^2} - \frac{GMm}{a(1 - e)}$$

$$E = -\frac{GMm}{2a}\,. \tag{18}$$

Thus we have an expression for the total energy of an elliptical orbit in terms of the semimajor axis a. If expression (18) is inserted into equation (17), equation (17) becomes

$$\dot{r}^2 + r^2\dot{\theta}^2 = v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

which is equation (8) given in section 308.

311. APPLICATIONS

At this point, problems will be given to illustrate the application of the expressions obtained for energy, period, velocity, etc. (These applications are based on problems prepared by Captain Frank B. Andrews, USN, Head of Science Department, United States Naval Academy.)

Application #5:

Assume that the satellite ECHO has a mass of 166 pounds and an orbital period of 117 minutes. Given that the mass of the Earth is $M_e = 5.98 \times 10^{24}$ kilograms, r_e = 3,444 nautical miles = 6.37×10^3 km, and that $G = 6.67 \times 10^{-11}$ $m^3/kg\ sec^2$, calculate the semimajor axis a.

$$T^2 = \frac{4\pi^2 a^3}{G\,M_e}$$

$$a = \frac{(117^2)\ (6.67)\ (5.98 \times 10^{13})}{4\pi^2}^{1/3}$$

= 7,950 km = 4,930 statue miles

= 4,282 nautical miles.

Application #6:

Calculate a for T = 116 minutes.

$$\left(\frac{117}{116}\right)^2 = \left(\frac{7950}{a_{116}}\right)^3$$

$$a_{116} = \left(\frac{116}{117}\right)^{2/3}\ (7950)$$

a_{116} = 7,880 km

= 4,255 nautical miles = 4,890 statue miles.

Application #7:

Given that perigee and apogee were observed to be 499 nautical miles and 1,177 nautical miles respectively, calculate e.

$$\begin{array}{r} 1177 \\ \underline{3444} \\ 4621 = R_{max} \end{array} \qquad\qquad \begin{array}{r} 499 \\ \underline{3444} \\ 3943 = R_{min} \end{array}$$

$$a = \frac{R_{max} + R_{min}}{2} = 4282 \text{ (as above)}$$

$$c = R_{max} - a = 4621 - 4282 = 339$$

$$e = \frac{c}{a} = \frac{339}{4282} = 0.0792\,.$$

Application #8:

Calculate the velocity at apogee and perigee for the T = 117 minutes orbit.

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right).$$

For apogee,

$r = R_{max}$ = 4,621 N. miles = 8,550 km.

$$v^2 = 6.67\ (5.98)\left(\frac{2}{.8550} - \frac{1}{.7950}\right)(10^6)$$

$v^2 = 43.1 \times 10^6\ m^2/sec^2$

v = 6.57 km/sec = 4.07 mi/sec

= 3.54 N. mi/sec = 12,700 kt.

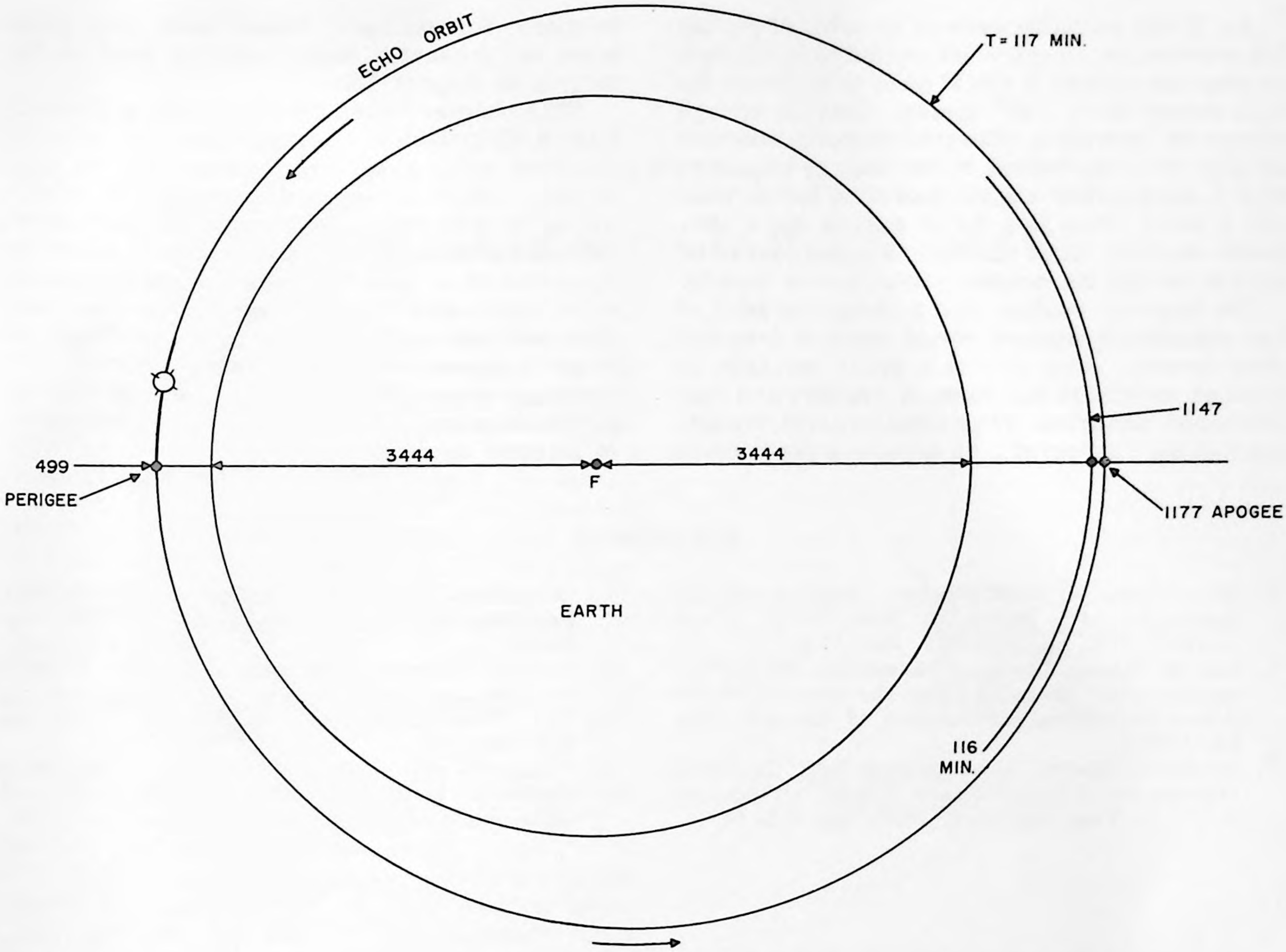

Figure 311.–Orbit of ECHO satellite on 5 January 1961. Path of T = 116 min. is not shown to scale, since it would appear extremely close to the T = 117 min. orbit in this drawing.

For perigee,

$$r = R_{min} = 3{,}943 \text{ N. miles} = 7{,}300 \text{ km.}$$

$$v^2 = 6.67\ (5.98)\left(\frac{2}{.7300} - \frac{1}{.7950}\right)(10^6)$$

$$= 59.0 \times 10^6 \text{ m}^2/\text{sec}^2$$

$$v = 7.68 \text{ km/sec} = 4.77 \text{ mi/sec}$$
$$= 4.15 \text{ N. mi/sec} = 14{,}900 \text{ kt.}$$

As was expected, the velocity at perigee is greater. However, since the eccentricity is small (e = 0.0792), the difference in velocities is not large (14.5%).

Application #9:

Find the difference in total energy between the T = 117 min. and the T = 116 min. orbits.

The total energy is given in expression (18) as

$$E = -\frac{GMm}{2a}$$

so, for the 117 min. orbit, a_{117} = 7,950 km. The mass of the satellite is 116 lbs. = 75.3 kg.

$$E_{117} = -\frac{6.67\ (5.98)\ (75.3)\ (10^{10})}{2\ (7950)}$$

$$= -\ 2.51 \times 10^9 \text{ joules.}$$

For the 116 min. orbit, a_{116} = 7,880 km.

$$E_{116} = -\frac{6.67\ (5.98)\ (75.3)\ (10^{10})}{2\ (7880)}$$

$$= \frac{7950}{7880}(E_{117})$$

$$= -\ 2.53 \times 10^9 \text{ joules.}$$

So, if the satellite were in an orbit of period 116 minutes, to increase its period of revolution by only one minute it would have to increase its total energy by 2×10^7 joules. This is enough energy to operate a standard washing machine all day; it is equivalent to the energy produced by a 5 horsepower engine operating for an hour and a half! This is a lot of energy for a 166-pound satellite to be carrying around just to be able to change its orbital period by one minute.

We begin to realize that a change in orbit of any significant amount would demand tremendous forces. This means a great increase in payload weight in the form of engines and fuel delivered into orbit. Thus actual control or guidance of the spacecraft, to actuate a rendezvous in space, for example, would pose many problems at present. More will be said on this subject in chapter 10.

This chapter began by first stating Newton's Law of Gravitation. Energy concepts were introduced next, using circular orbits for simplicity. After a brief discussion of conics, Kepler's laws were presented and then proofs for these laws were given. Then, using the equations of motion, the general energy expressions applicable to all conics were obtained. Also included were two sections of "applications" designed to emphasize the general relationships presented. The next chapter will apply these principles in a more specific analysis of satellite orbits.

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CHAPTER 4

DETERMINATION OF SATELLITE ORBITS

It has been pointed out in the preceding chapter that the equations of motion describing the two-body problem can be expressed in several ways, and several methods are available for solving these equations. But in each case a number of parameters, whose values will specify some particular orbit, appear in the results in the form of constants of integration. The actual geometric or physical meaning of these parameters, or elements of the orbit, will depend upon the method of approach used in solving the Kepler problem.

401. FORMS OF THE CONSTANTS OF INTEGRATION

Though the exact number of elements necessary is sometimes confused by a particular approach, six constants will always be needed to determine a specific conic section in space and locate the satellite body within this orbit. However, just any six might not be sufficient. This fact is in agreement with the well-known result of classical mechanics: if the force laws which govern the motion of a particle are known exactly, and if the position and velocity of the particle are known at any instant of time, as shown in figure 401A, then the motion of the particle is completely defined for all time, prior and subsequent. Since both position and velocity are vector quantities, then the knowledge of each implies the knowledge of three bits of information, in three-dimensional space at least, so six parameters are thus given.

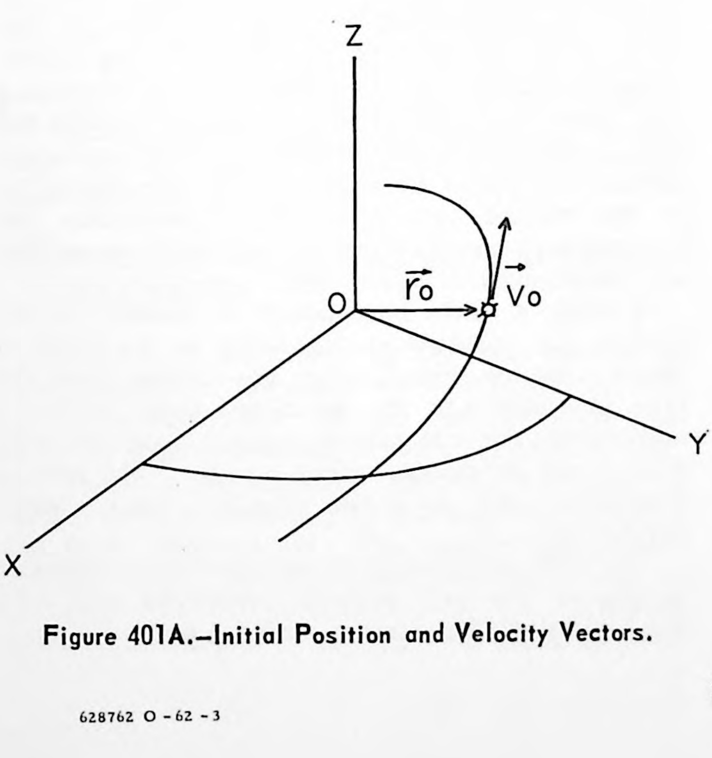

Figure 401A.—Initial Position and Velocity Vectors.

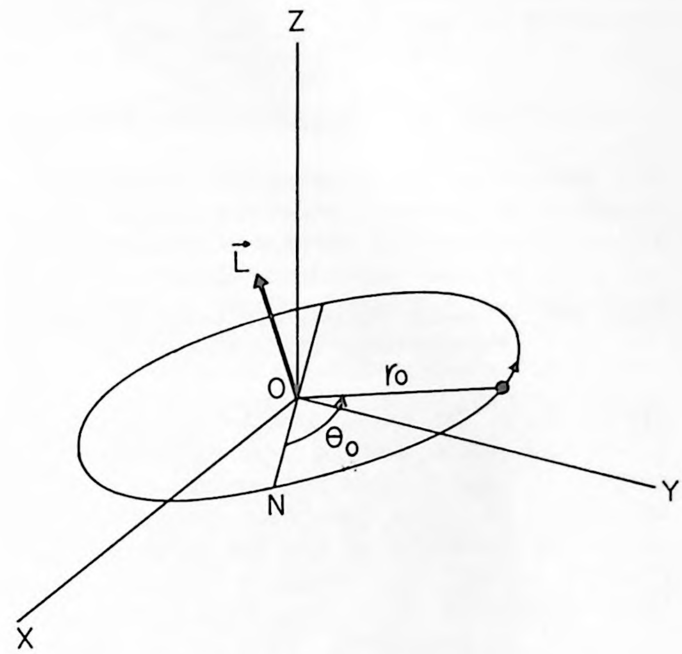

Figure 401B.—Angular Momentum and Initial Position within Orbital Plane.

Rather than specify some initial position and velocity vector for the satellite, one can just as well give the energy and angular momentum of the body and its initial position within the plane of its orbit (two coordinates). This may appear at first to be only four quantities, but it should be remembered that angular momentum is a vector quantity; so its three components must be given, and again six quantities result. This is illustrated in figure 401B.

Another alternative set of orbital elements is formed by the three-direction cosines of the

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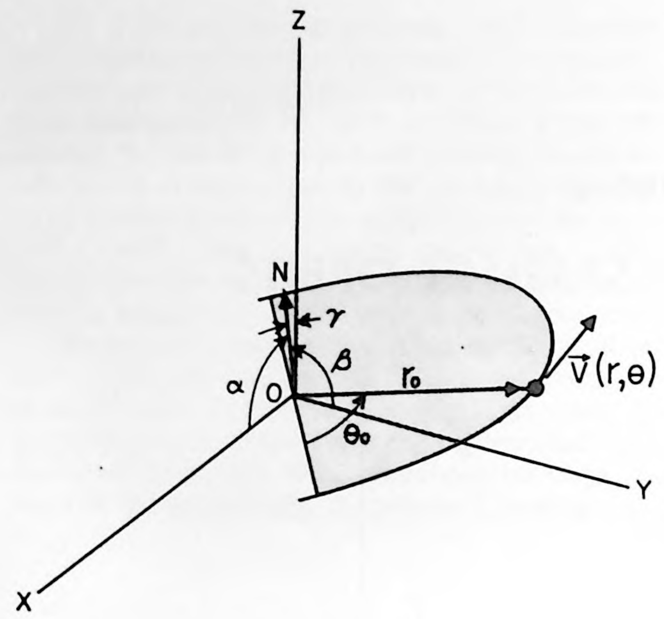

Figure 401C.–Direction Cosines and Initial Position and Velocity within Orbital Plane.

orbital plane, the initial position of the satellite within this plane, and the first derivatives with respect to time of these two coordinates. If this appears to be seven quantities instead of six, it should be remembered that the three-direction cosines satisfy the condition that the sum of their squares be unity; thus when two are given the third is redundant. This set is shown in figure 401C.

While each of the sets of six parameters described above[1] are sufficient to define an unperturbed conical orbit in space, and locate the body in this orbit, none of these are commonly used. However, they can be converted into the preferred set of elements, and this conversion process is the heart of the problem. The preference for the set which will now be described is based on convenience and tradition—the astronomers have been doing it for years.

402. PREFERRED ORBITAL ELEMENTS

Two quantities specify a given conic, as can be seen from the general equation

$$r = \frac{p}{1 + e \cos \theta}$$

where the semiparameter or semilatus rectum p determines the size and the eccentricity e determines the general shape of the curve. The variables r and θ are focal polar coordinates with origin at the central body; r is called the radius and θ the true anomaly. If we restrict ourselves to elliptical orbits, it is more common to give the semimajor axis a and the eccentricity. One could give the semiminor axis b, or the center to focus distance c instead of the eccentricity, for these are all related by the equations

$$e = \frac{c}{a} = \sqrt{1 - \frac{b^2}{a^2}} .$$

Two angles are necessary to specify the orientation of the orbital plane to some reference XYZ coordinate system with origin at the central body. The preferred elements are the dihedral angle between the orbital plane and the XY plane, called the inclination i, and the angle in the XY plane from the positive X axis around to the line of intersection of the two planes where the satellite rises above the XY plane. This line of intersection of the two planes is called the line of nodes, while the angle is called, in general, the longitude of the ascending node Ω.

A third angle is necessary to fix the orientation of the orbit within the orbital plane. Since the periapsis, or point of closest approach of the satellite to its principal focus, lies on the major axis of the conic, or along the direction for which the true anomaly $\theta = 0$, it is convenient to use this as the reference point.

The angle measured in the orbital plane from the ascending node up to this periapsis point, in the direction of the satellite's motion, uniquely fixes the orientation of the conic finally to the reference XYZ system. It is usually called the argument of periapsis ω. (Note: Periapsis is the general name. If the central body is the Sun, the term perihelion is used; if it is the Earth, then perigee.) It is convenient at this point to define another angle used later in this chapter. The sum of the argument of periapsis and true anomaly is the angle measured in the orbital plane from the ascending node up to the actual position of the satellite, in the direction of its motion, and it is called generally the argument of the latitude u. All of these angles are illustrated in figure 402.

Finally a sixth parameter is needed to establish the position of the body in its orbit at some time, or epoch. One can either give the true anomaly θ at the specified time, or more convenient, one can give the exact time the body was at the periapsis where $\theta = 0$. This choice simplifies one equation slightly, as we shall see later.

In brief summary, these are the preferred forms of the six orbital elements for any Keplerian orbit about any central body:

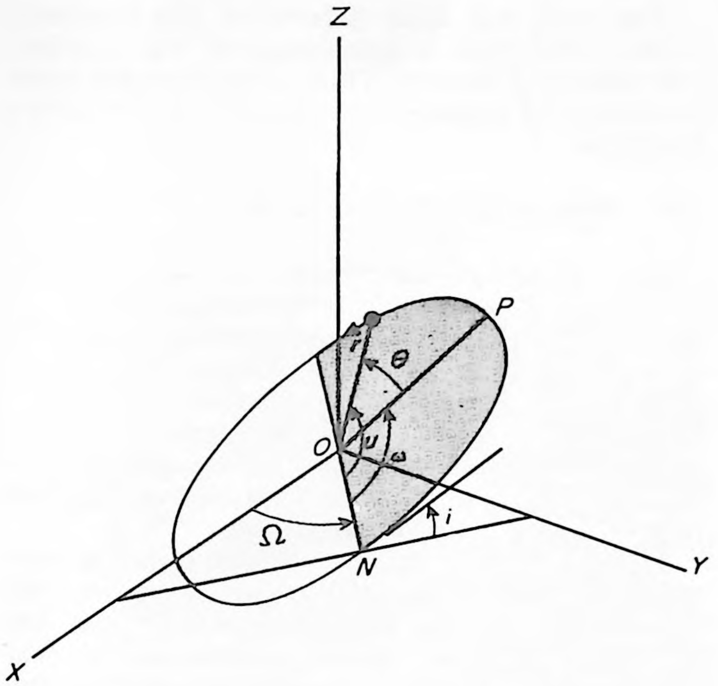

Figure 402.—Angles Used to Specify the Orbit.

- a the semimajor axis, or p the semiparameter,
- e the eccentricity,
- i the inclination of the orbital plane,
- Ω the longitude of the ascending node,
- ω the argument of the periapsis, and
- t_p the time at periapsis.

It should be noted at this point that for parabolic orbits $e = 1$, so that the five remaining elements are sufficient. For circular orbits $e = 0$, and the argument of periapsis has no meaning since the radius is constant. Therefore, the other four elements are sufficient. But on the other hand, one usually does not know that an orbit is circular or parabolic before he finds the elements from observational data. In this sense it is always necessary to find all six parameters.

403. REQUIREMENTS FOR DETERMINING ELEMENTS

Let us now consider the type and number of observations of the satellite necessary and sufficient to determine all the orbital elements. By the ordinary methods of positional astronomy, two angular coordinates defining the line of sight from the observer to the body are obtained with each observation. The specific coordinate systems used, and the transformations one to another, will be considered in the following chapter. Suffice it to say that one can determine from this observational data the angular coordinates of the satellite relative to the center of the Earth, or to its own central body if different from the Earth. Obviously then, three such measurements of two quantities each will be necessary to determine the six elements of the orbit. If the observations are carefully selected to avoid double solutions, three will be sufficient to determine an unperturbed conical orbit. If the individual measurements can be made very accurately, all three observations can be taken while the satellite traverses only a very small portion of its orbit.

404. THE COMPUTATION OF ORBITS

The actual computation of the orbital elements, which is a basic problem in positional astronomy, can be divided into two major areas—the first orbit determination and the orbit improvements.[2] The first determination is merely the calculation of the six elements from a minimal set of data, as described above, for a completely unknown orbit. Orbit improvements consider the data from many observations of the body, and seek to refine the elements of the first determination such that the analytical orbit fits the observed orbit as closely as possible.

The problem of a first orbit determination from only three observations was first solved by Newton for the special case of a comet travelling in a parabolic path, using graphical methods. The first complete solution by analytical methods was given by Euler in 1744. Lagrange wrote several papers on orbit theory, but his methods are not used in practice today. Laplace published a new method in 1780 which is the basis for many of the present day orbit determinations. Finally the great German mathematician Gauss conceived of another method at the age of 24, given impetus by the discovery and subsequent loss of the first minor planet, Ceres, in 1801.

The classic methods of Gauss and Laplace will now be described briefly. Inasmuch as both methods were intended for the determination of orbital elements for planets of the Sun as observed from the Earth, they necessarily involve some coordinate transformations, and assume a knowledge of the Sun's location relative to the Earth at the times of the observations. Obviously one must, in the final analysis, know some distance in order to determine an actual distance from earth to satellite, for example. If only angular quantities are known, then all distances can be found only relative to each other. In actual practice of determining orbits of the Sun's planets, the reference length is the mean distance

of the Earth from the Sun, defined as one astronomical unit. Its value in terms of Earth-bound standards of length such as the meter is known only approximately. Later we shall see that the problem of orbit determination is considerably simplified when we restrict ourselves to satellites of the Earth as observed from the Earth.

405. THE METHOD OF LAPLACE

The Laplacian method centers around the differential equations of motion of the satellite about its central body, and seeks to determine the position and velocity vectors of the satellite at one instant of time. This information is then transformed into the standard orbital elements. For each observation the two angular coordinates are reduced by standard techniques to qualities we shall for now call only the latitude b and longitude ℓ.

Other names will be applied in the following chapter where we shall consider some specific coordinate systems. From these two quantities the direction cosines λ, μ, ν are determined using the equations

$$\lambda = \cos b \cos \ell$$

$$\mu = \cos b \sin \ell$$

$$\nu = \sin b.$$

The rectangular coordinates x, y, z of the Sun with respect to the Earth are usually obtained from the American Ephemeris and Nautical Almanac.

In the first step the values of the first and second time derivatives of λ, μ, ν, x, y, z are determined at a time near to one of the observations, say at the middle one of the three. The second step imposes the conditions that the unknown body follow the equations of motion in its path around the Sun, and likewise around the Earth. Each case is assumed to be a two-body problem, the attractions of other bodies being ignored. The result is a set of equations relating the unknown distance of the body from the Earth, its first and second time derivatives, and the unknown distance of the body from the Sun to the known quantities λ, μ, ν, x, y, z.

The third step is to determine these unknown distances from the set of equations in the preceding step and from the additional condition that the Earth, Sun, and unknown form a triangle. The result is the coordinates of the unknown body relative to the Sun. In the following step the three components of the velocity of the unknown body are determined.

The last step is to determine the elements of the orbit from a knowledge of the position and velocity vectors. This completes the brief summary[3] of Laplace's method of orbit determination.

406. THE METHOD OF GAUSS

The Gaussian method can be described as based upon the integral of the equations of motion, and similarly requires only two polar coordinates for three observations, the times, and the positions of the Sun relative to the Earth. The general plan is to determine three coordinates of the unknown body at two times, from which the orbital elements can be determined using equations Gauss developed.

The first step requires that the unknown body move in a plane containing the center of the Sun. The next step imposes the condition that the body follow the law of gravitation, or that the sectors of the body's orbit between the first and second, and between the second and third radius vectors, have areas in accordance with Kepler's second law. This is done by integrating the equations of motion as a power series in the time intervals.

In the third step equations are developed to determine the distance of the unknown body from the Earth at each of the times of observation. Again the triangle condition is used to relate the distances of the unknown body from Earth and Sun to the known value of Earth-Sun distance. The next step consists of solving these equations and finding the three-dimensional Sun-centered coordinates of the body at two of the times of observation.

The final step involves the actual determination of the set of orbital elements from these two positions in the orbit. Obviously, if one were given this information at the outset, all of the lengthy procedures described above could be eliminated. In chapters 6 and 7 methods will be illustrated which give all three coordinates of an earth satellite. Anticipating the ability to find this information for two positions of the satellite, let us now consider in detail this final step of the Gaussian method, so that we will be able to determine the orbital elements of our earth satellite when the need arises.

407. SOLUTION FOR ELEMENTS GIVEN THREE-DIMENSIONAL POSITIONS

We will assume that three-dimensional fixes are given for the satellite at two instants of time, in the form of two angular coordinates

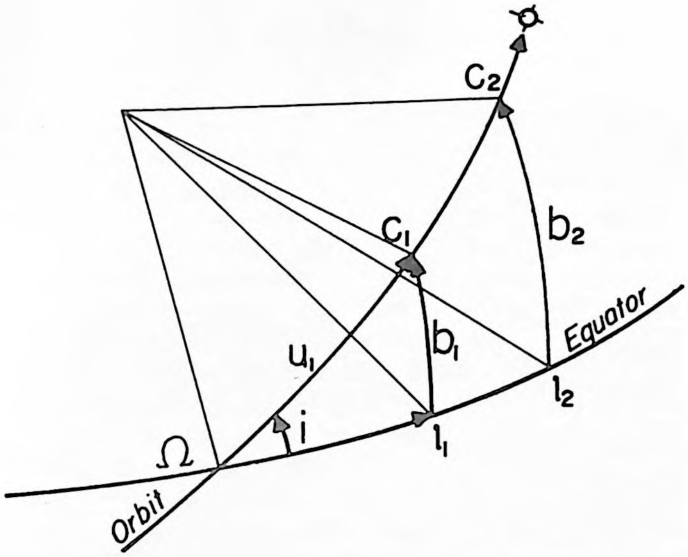

Figure 407A.—Spherical Triangle relating b, l, u, and inclination.

and the radial distance from the center of attraction. For the sake of generality we still call the angular coordinates simply latitude b and longitude ℓ. Where C_1 and C_2 are the known positions of the body at the times t_1 and t_2, the angular coordinates are as shown in figure 407A.

The inclination angle and longitude of the ascending node can be determined immediately. It follows from the spherical triangles $C_1 \Omega \ell_1$ and $C_2 \Omega \ell_2$ that

$$\tan i \sin(\ell_1 - \Omega) = \tan b_1,$$

$$\tan i \sin(\ell_2 - \Omega) = \tan b_2.$$

Since $\ell_2 - \Omega = (\ell_2 - \ell_1) + (\ell_1 - \Omega)$, the equations can be rewritten

$$\tan i \sin(\ell_1 - \Omega) = \tan b_1,$$

$$\tan i \cos(\ell_1 - \Omega) = \frac{\tan b_2 - \tan b_1 \cos(\ell_2 - \ell_1)}{\sin(\ell_2 - \ell_1)},$$

which determine i and Ω uniquely. The inclination is less or greater than 90° according as ℓ_2 is greater or less than ℓ_1.

The angle along the orbit from the node to the body, or the argument of the latitude u, also can be obtained from the information given. It follows from the figure above that

$$\cos(\ell_j - \Omega) \cos b_j = \cos u_j,$$

$$\sin(\ell_j - \Omega) \cos b_j = \sin u_j \cos i,$$

$$\sin b_j = \sin u_j \sin i, \qquad (j = 1,2)$$

which uniquely define u_1 and u_2. It should be noted that all of these angles could be determined most easily by spherographical methods on a globe such as that of McMillen, if very precise answers are not as important as speed and convenience. One only need plot the two positions, given the angular coordinates, and connect them with a great circle extended through the reference or equatorial plane. A spherical protractor is necessary then to measure the inclination, the longitude of the node can be read directly if the equator is scaled, and the arguments of the latitude can be measured by appropriate dividers.

There remain the other four orbital elements to be determined from the given data, and to complete the task we must use some rather involved equations derived from Gauss. Only the results of that derivation[3] will be given here.

Let us define the ratio of the area of the sector to that of the triangle between the two radius vectors r_1 and r_2 as η.

$$\eta = \frac{\text{Sector}}{\text{Triangle}} = \frac{k\sqrt{p}\,(t_2 - t_1)}{r_1 r_2 \sin(u_1 - u_1)}$$

where p is the semiparameter of the conic, and k is the force constant appearing in the equations of motion. Further, let

$$2f = u_2 - u_1,$$

$$m = \frac{k(t_2 - t_1)}{(2 r_1 r_2 \cos f)^{3/2}},$$

$$L = \frac{r_1 + r_2}{4 r_1 r_2 \cos f} - \frac{1}{2},$$

and

$$2g = E_2 - E_1$$

where the E's are the eccentric anomalies. Unlike true anomaly, an eccentric anomaly of an ellipse is measured at the center of the ellipse, and is defined as the angle from periapsis to the intersection of a line drawn through the satellite perpendicular to the major axis with a circle which has the major axis as a diameter. Reference to figure 407B will clarify this definition.

The two equations developed by Gauss can then be written as

$$\eta^2 = \frac{m^2}{L + \sin^2 \frac{1}{2} g},$$

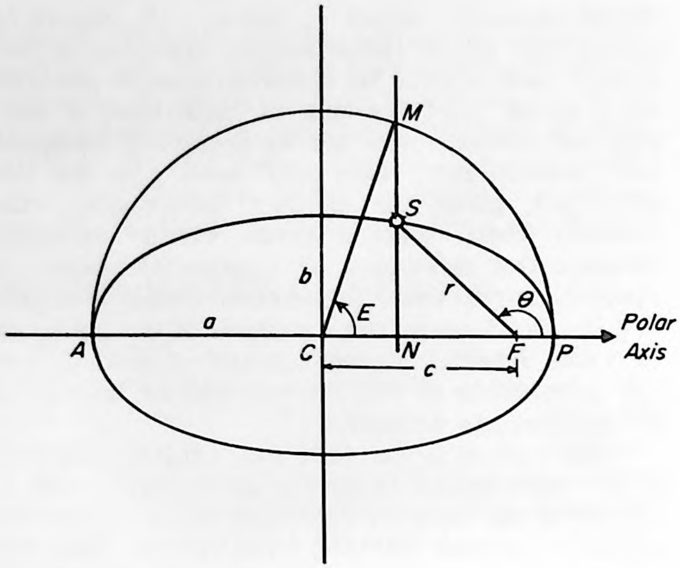

Figure 407B.–Relation between True and Eccentric Anomaly.

and

$$\frac{\eta^3 - \eta^2}{m^2} = \frac{2g - \sin 2g}{\sin^3 g}$$

in which η and g are the unknowns. These two equations are then solved simultaneously for η, by letting

$$q = \sin^2 \frac{1}{2} g,$$

$$\xi = \frac{2}{35} q^2 + \frac{52}{1575} q^3 + \cdots ,$$

and

$$h = \frac{m^2}{\frac{5}{6} + L + \xi}.$$

By these substitutions and the elimination of g between the two Gaussian equations, a cubic equation for η results with the form

$$\eta^3 - \eta^2 - h\eta - \frac{h}{9} = 0.$$

This equation can be solved for η to any desired degree of accuracy by an iterative process. If $(1/2)g$ is fairly small, then q is a small quantity of the second order, and ξ is of the fourth order. In the first approximation one assumes ξ is zero and finds h. This value is substituted into the cubic and the real position root for η is determined. From the first of the Gauss equations it follows that

$$q = \frac{m^2}{\eta^2} - L.$$

The first approximation for η is substituted in this equation to find q, and this value of q is used to determine ξ from the power series above. With this approximation for ξ an improved value for η is then calculated, and the process is repeated to any desired accuracy. Experience shows that this method of computing the ratio of the orbit sector to the triangle converges very rapidly, even when the time interval, and hence the value of g, is fairly large. Tables to facilitate the solution are given in Watson's Theoretical Astronomy[4] for η with the argument h, and for ξ with the argument q. These greatly reduce the computational work.

At this point the type of conic section has been determined according as q is positive, zero, or negative, the orbit will be an ellipse, a parabola, or a hyperbola. This follows from the fact that

$$q = \sin^2 \frac{1}{2} g = \sin^2 \frac{1}{4} (E_2 - E_1),$$

and the eccentric anomalies are real in ellipses, zero in parabolas, and imaginary in hyperbolas.

After g has been determined by this procedure, it is a simple matter to find the remaining orbital elements. The semimajor axis is determined from the equation

$$a = \frac{r_1 + r_2 - 2\sqrt{r_1 r_2} \cos g \cos f}{2 \sin^2 g}.$$

The semiparameter p is given by rearranging the equation defining η,

$$p = \left[\frac{\eta r_1 r_2 \sin 2f}{k(t_2 - t_1)}\right]^2,$$

and hence the eccentricity of the conic can be found using

$$p = a(1 - e^2) \text{ for an ellipse, or}$$

$$p = a(e^2 - 1) \text{ for a hyperbola.}$$

$$(e = 1 \text{ for a parabola.})$$

The argument of the latitude u, measured from the ascending node along the orbit to the body, has been determined above at each of times of observation. The true anomaly of the body at either of the times can be computed from the familiar polar equation of the conic,

$$r = \frac{p}{1 + e \cos \theta},$$

and since this angle originates from the periapsis point we can easily determine the argument of the periapsis by

$$\omega = u - \theta.$$

Finally, the time of periapsis passage t_p can be found from several equations,[2,3] depending upon the type of conic section.

Parabola:

$$k(t - t_p) = \frac{1}{2} p^{3/2} \left[\tan \frac{1}{2}\theta - \frac{1}{3} \tan^3 \frac{1}{2}\theta \right].$$

Ellipse:

Eccentric anomaly is determined from

$$\tan E = \frac{\sqrt{1 - e^2} \sin \theta}{e + \cos \theta},$$

after which Kepler's equation

$$E - e \sin E = \frac{2\pi}{T}(t - t_p) = \frac{\sqrt{k}}{a^{3/2}}(t - t_p)$$

yields the value of t_p. T is the anomalistic period of the body.

Hyperbola:

The parameter F is defined by

$$\tanh \frac{1}{2} F = \sqrt{\frac{e - 1}{e + 1}} \tan \frac{1}{2}\theta$$

after which t_p is given by

$$-F + e \sinh F = \frac{k\sqrt{1 + m}}{a^{3/2}}(t - t_p).$$

This completes our brief survey of the methods for determining orbital parameters of a satellite from observed positional data. The reverse problem of predicting positions of a satellite at some particular time, given its orbital elements, is another of the basic problems of positional astronomy. The technique of computing an ephemeris, as such a table of predicted coordinates is called, will be considered in chapter 8.

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CHAPTER 5

COORDINATE SYSTEMS

501. REFERENCE SYSTEMS

To define an orbit in space or a position in an orbit a reference system is necessary. All positions being relative, the common practice is to relate a position (in space) to a convenient coordinate system that moves with the observer. In defining such a coordinate system an origin and a reference plane, the XY plane, are selected.

502. GALACTIC SYSTEM

If the origin lies at the center of the galaxy and the galactic plane is used as the reference plane with an arbitrary direction chosen to orient it, the system is called a galactic system. This system, however, has no practical application to space navigation at present.

503. HELIOCENTRIC ECLIPTIC SYSTEM

If the center of the Sun lies at the origin, the system is called heliocentric. And if the plane of the Earth's orbit defines the reference plane, it is called heliocentric ecliptic. The illustration (figure 503) shows how a body's position is defined by means of heliocentric longitude ℓ measured eastward from the vernal equinox, heliocentric latitude b and radius r from the origin.

504. GEOCENTRIC SYSTEMS

With the origin lying at the center of the Earth, as would be used for close-in terrestrial satellites, a geocentric system is obtained. Two geocentric coordinate systems are called equatorial or ecliptic depending on whether the plane of the equator or the plane of the ecliptic is used as the respective reference plane. Right ascension α and declination δ are the two angles used in the geocentric equatorial system for locating, in space, a line of bearing from the origin. Sidereal hour angle (SHA) is used in place of α for navigation purposes. They are shown in figure 504A. It can be seen that right ascension is measured eastward (SHA westward) from the vernal equinox and declination measured north or south of the equatorial plane. The third component, the distance from the origin, remains r.

When the ecliptic plane is the reference plane in a geocentric system the three coordinates become longitude λ measured eastward from the vernal equinox, latitude β measured from the ecliptic plane and r, the distance from the center.

505. HORIZON SYSTEMS

Another coordinate system deserves mention. The topocentric horizon system (observer is at the origin of the coordinate system) used for tracking purposes, is illustrated in figure 505. Here the coordinates become altitude H above the horizon, azimuth Az measured from true north and ρ, distance from the observer. A horizon system with the center of the Earth as the origin is called a geocentric horizon system.

506. TRANSFORMATIONS

To define an orbital plane in space relative to any coordinate system, the longitude of the ascending node Ω and the inclination i are used; to orient the orbit in the orbital plane, longitude of periapsis ω is needed.

In order to transform from one coordinate system to another, translations and rotations are performed in sequence. Rectangular coordinates simplify the translation operation, while polar coordinates reduce the messiness of a rotation. A transformation from an arbitrary XYZ system to a PLP' system (figure 506), where the origin remains identical (no translation), is defined by

$$P = X \cos\omega \cos\Omega - X \cos i \sin\Omega \sin\omega + Y \cos\omega \sin\Omega + Y \cos i \cos\Omega \sin\omega + Z \sin\omega \sin i \quad (1)$$

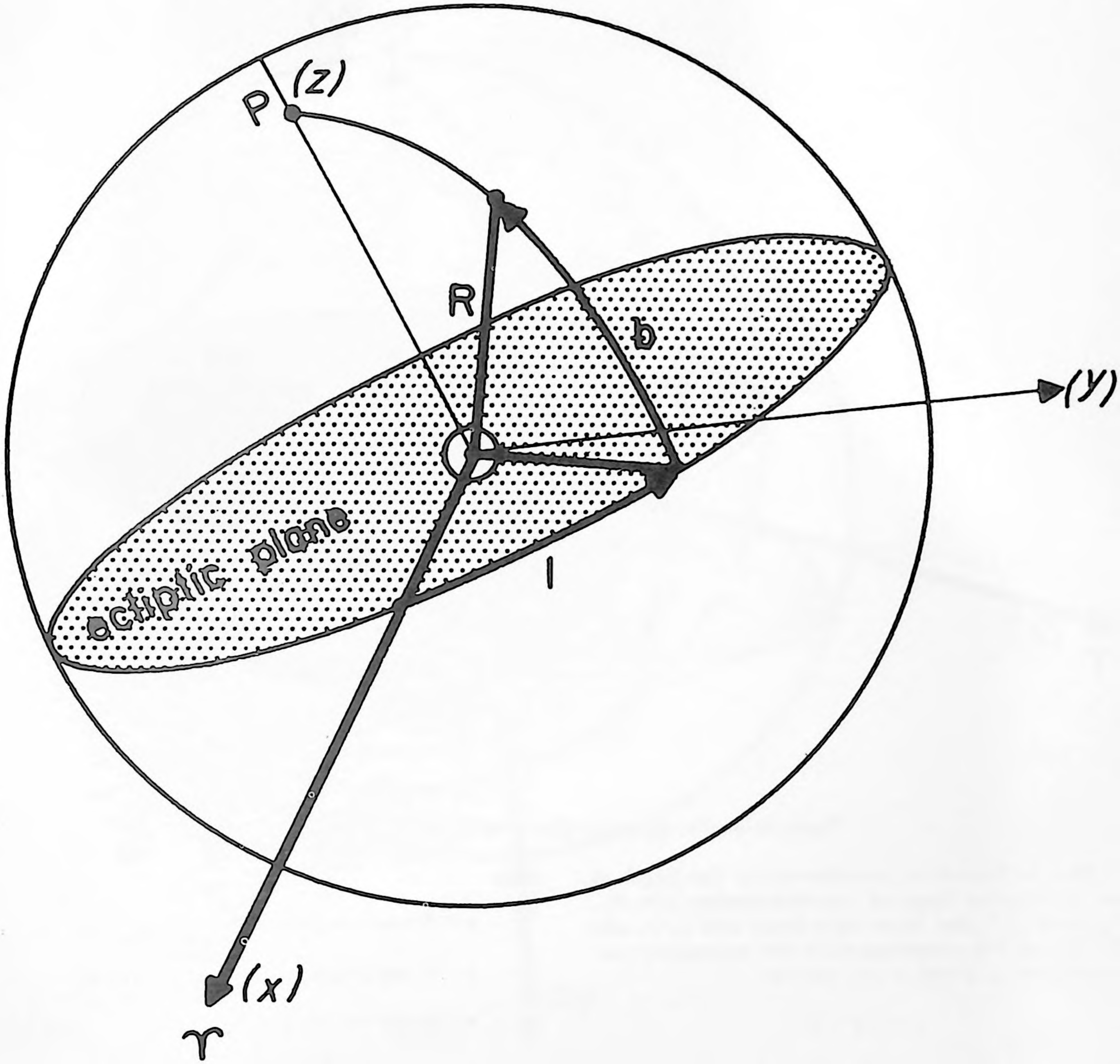

Figure 503.–The Heliocentric Ecliptic Coordinate System.

$$L = -X \sin\omega \cos\Omega - X \cos i \sin\Omega \cos\omega - Y \sin\omega \sin\Omega$$

$$+ Y \cos i \cos\Omega \cos\omega + Z \cos\omega \sin i \qquad (2)$$

$$P' = X \sin i \sin\Omega - Y \sin i \cos\Omega + Z \cos i. \qquad (3)$$

These equations come about by performing in succession three separate rotations about the Z axis, about the new P axis and about the new P' axis.

507. TRANSFORMATION FROM HELIOCENTRIC ECLIPTIC TO GEOCENTRIC ECLIPTIC

Since this transformation involves only a translation of the origin from the center of the Sun to the center of the Earth, polar coordinates are changed to rectangular coordinates by

$$X = R \cos b \cos \ell, \qquad Y = R \cos b \sin \ell, \qquad Z = R \sin b.$$

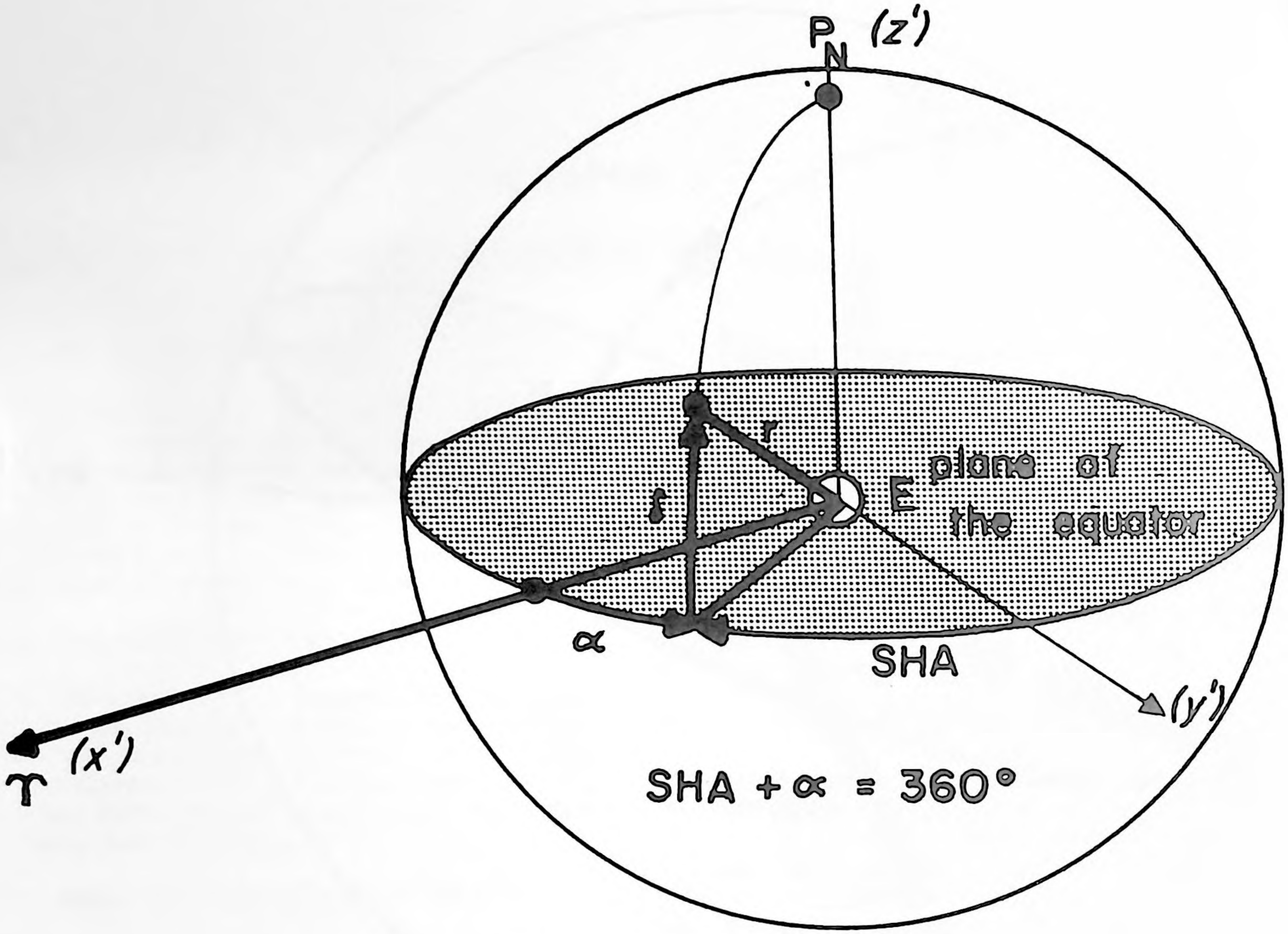

Figure 504A.–The Geocentric Equatorial Coordinate System.

The heliocentric coordinates of the Earth at the particular time of transformation are X_e, Y_e, and Z_e* and those of a body are X, Y, and Z. To get the coordinates in the geocentric coordinates x, y and z one writes

$$x = X - X_e$$

$$y = Y - Y_e$$

$$z = Z - Z_e$$

where

$$x = r \cos \beta \cos \lambda$$

$$y = r \cos \beta \sin \lambda$$

$$z = r \sin \beta$$

*It should be noted that b_e is rarely more than one second of arc and therefore $Z_e \sim 0$ which would make $z \sim Z$.

thus

$$x = R \cos b \cos \ell - R_e \cos b_e \cos \ell_e$$

$$y = R \cos b \sin \ell - R_e \cos b_e \sin \ell_e$$

$$z = R \sin b - R_e \sin b_e$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\beta = \tan^{-1}\left[\frac{z}{\sqrt{x^2 + y^2}}\right]$$

$$\lambda = \tan^{-1} \frac{y}{x} .$$

508. TRANSFORMATION FROM GEOCENTRIC ECLIPTIC TO GEOCENTRIC EQUATORIAL

Let x, y and z be the geocentric ecliptic coordinate of a body as before and x', y' and z'

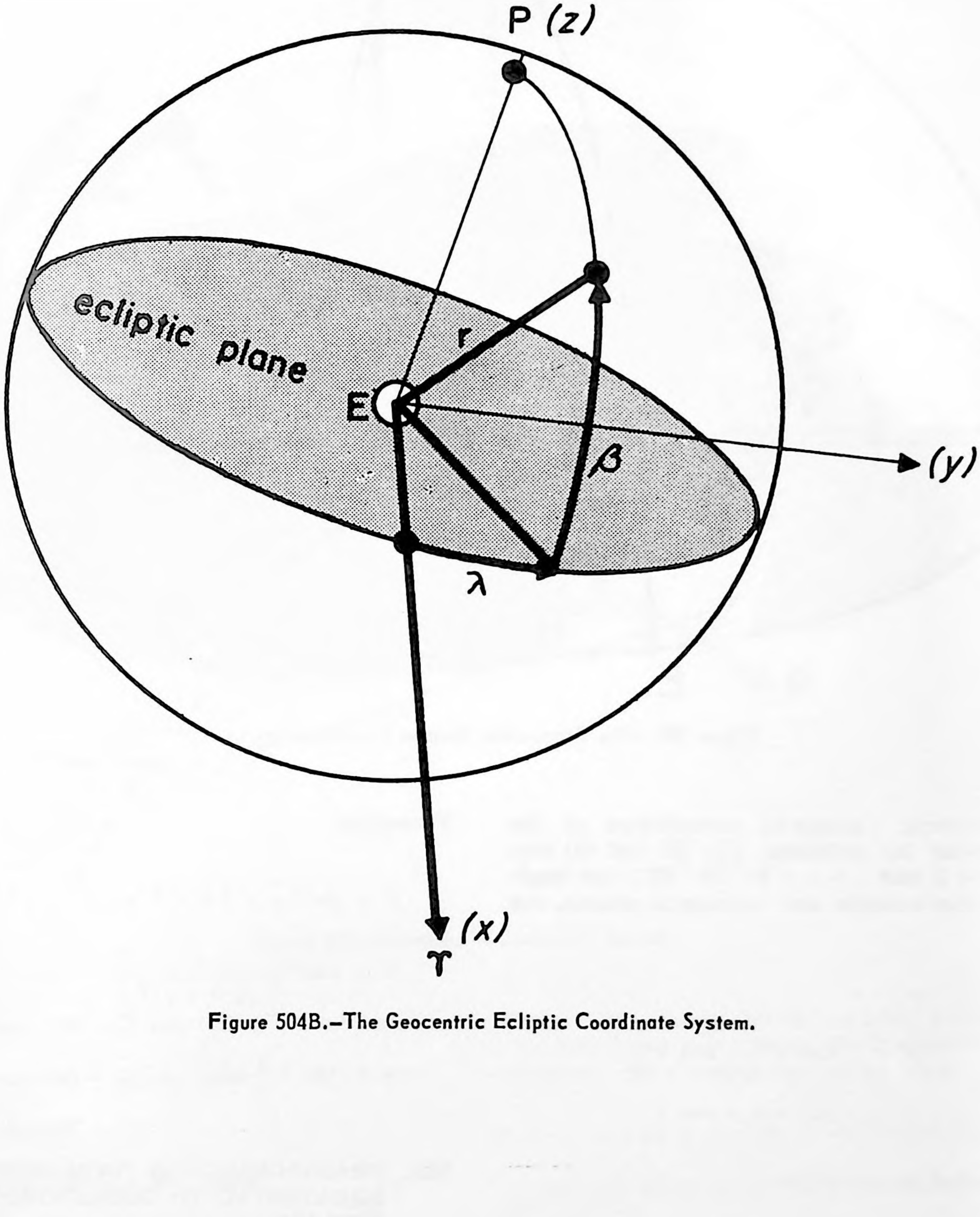

Figure 504B.–The Geocentric Ecliptic Coordinate System.

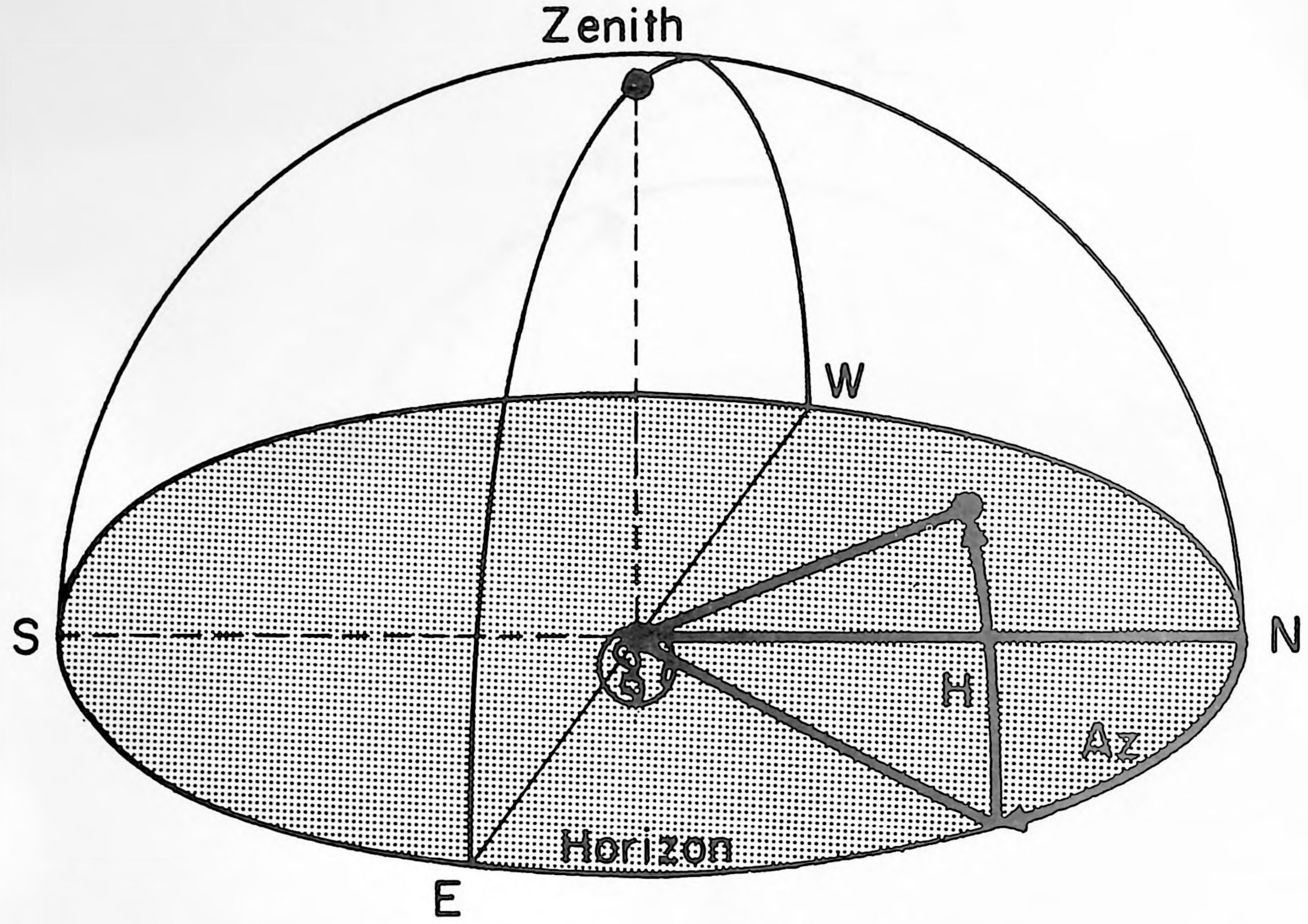

Figure 505.–The Topocentric Horizon Coordinate System.

the geocentric equatorial coordinates of the body. Using the relations (1), (2) and (3) with $\Omega = 0$, $\omega = 0$ and $i = \epsilon = 23° 26' 59''$, the angle between the ecliptic and equatorial planes, one has

$$x' = x$$

$$y' = y \cos \epsilon + z \sin \epsilon$$

$$z' = y \sin \epsilon + z \cos \epsilon$$

and in polar coordinates

$$x' = r \cos \beta \cos \lambda$$

$$y' = r \cos \beta \sin \lambda \cos \epsilon + r \sin \beta \sin \epsilon$$

$$z' = - r \cos \beta \sin \lambda \sin \epsilon + r \sin \beta \cos \epsilon .$$

Therefore

$$r = \sqrt{x'^2 + y'^2 + z'^2} = \sqrt{x^2 + y^2 + z^2}$$

$$\delta = \tan^{-1} \left[\frac{z'}{\sqrt{x'^2 + y'^2}} \right]$$

$$\alpha = \tan^{-1} \frac{y'}{x'} .$$

509. TRANSFORMATION FROM GEOCENTRIC EQUATORIAL TO GEOCENTRIC HORIZON

Again we use only equations (1), (2), and (3) because the origin is not shifted. For the rectangular coordinates in the geocentric horizon system use N_e, W_e, and Z_e. Hence

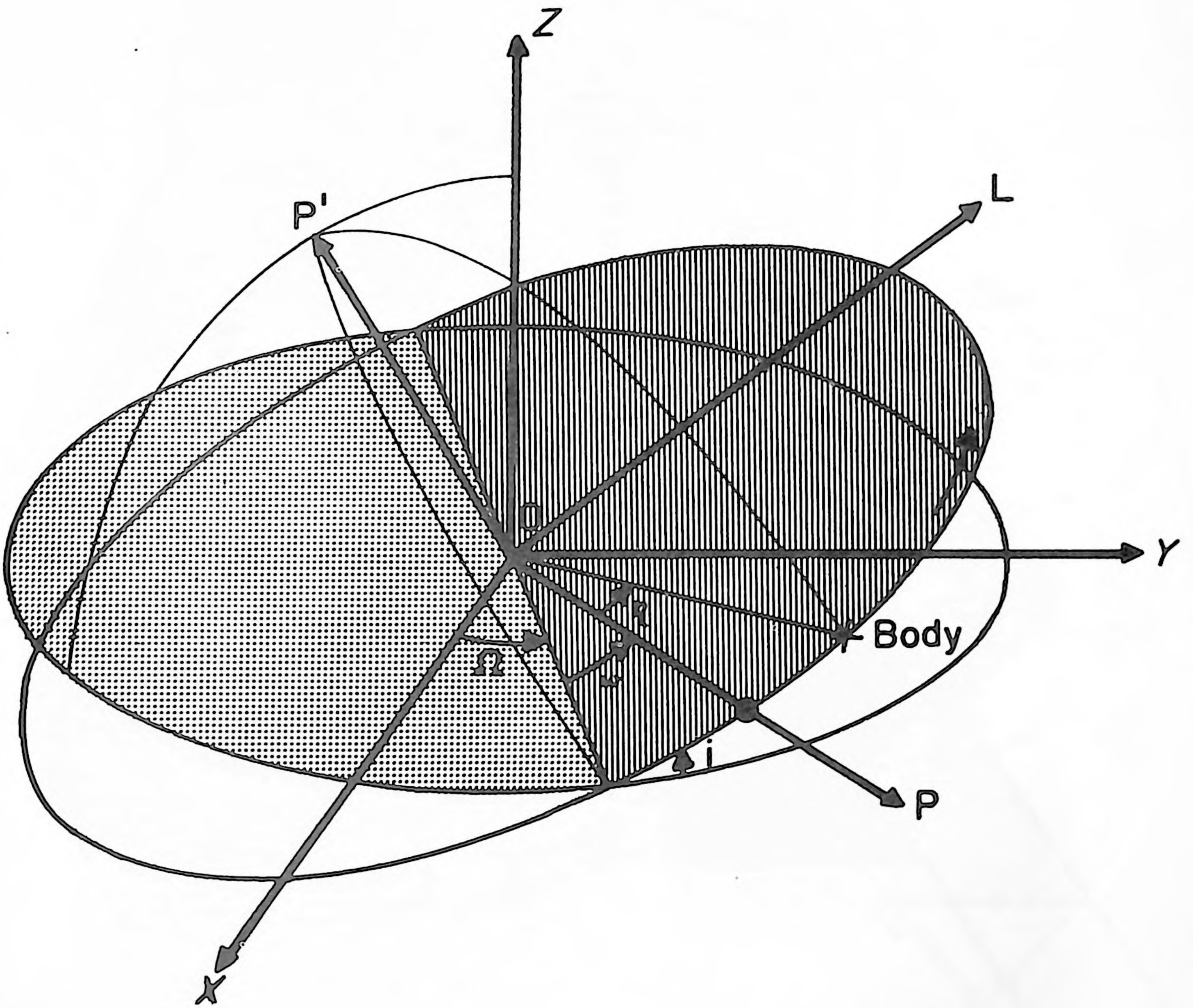

Figure 506.–Rotations of Coordinate System.

$$N_e = -\,x' \cos(90 - L)\sin(\text{GHA}\ \Upsilon + 90 - \text{Long}) + y'\cos(90 - L)\cos(\text{GHA}\ \Upsilon + 90 - \text{Long}) + z'\sin(90 - L)$$

$$W_e = -\,x'\cos(\text{GHA}\ \Upsilon + 90 - \text{Long}) - y'\sin(\text{GHA}\ \Upsilon + 90 - \text{Long})$$

$$Z_e = x'\sin(90 - L)\sin(\text{GHA}\ \Upsilon + 90 - \text{Long}) - y'\sin(90 - L)\cos(\text{GHA}\ \Upsilon + 90 - \text{Long}) + z'\cos(90 - L)$$

where L is latitude (north positive, south negative) and Long is longitude (east negative, west positive). Substituting into polar form

$$N_e = -(r\cos\delta\cos\alpha)\cos(90 - L)\sin(\text{GHA}\,\Upsilon + 90 - \text{Long}) + (r\cos\delta\sin\alpha)\cos(90 - L)\cos(\text{GHA}\,\Upsilon + 90 - \text{Long}) + (r\sin\delta)\sin(90 - L)$$

$$W_e = -(r\cos\delta\cos\alpha)\cos(\text{GHA}\ \Upsilon + 90 - \text{Long}) - (r\cos\delta\sin\alpha)\sin(\text{GHA}\ \Upsilon + 90 - \text{Long})$$

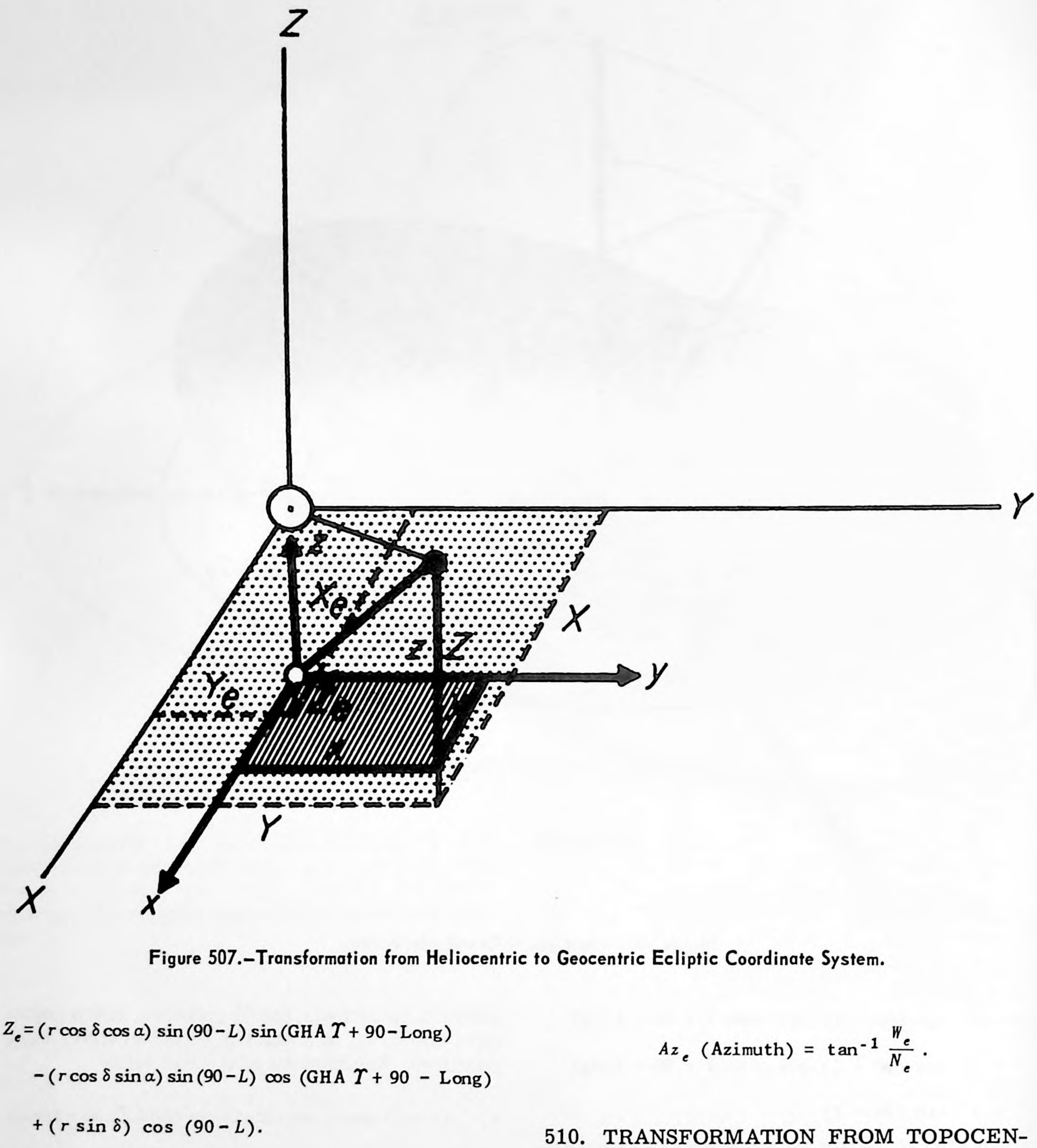

Figure 507.–Transformation from Heliocentric to Geocentric Ecliptic Coordinate System.

$$Z_e = (r \cos \delta \cos \alpha) \sin (90 - L) \sin (\text{GHA } \Upsilon + 90 - \text{Long})$$

$$- (r \cos \delta \sin \alpha) \sin (90 - L) \cos (\text{GHA } \Upsilon + 90 - \text{Long})$$

$$+ (r \sin \delta) \cos (90 - L).$$

It follows that

$$r = r \text{ (naturally)}$$

$$H_e \text{ (Altitude)} = \tan^{-1} \left[\frac{Z_e}{\sqrt{N_e^2 + W_e^2}} \right]$$

$$Az_e \text{ (Azimuth)} = \tan^{-1} \frac{W_e}{N_e} .$$

510. TRANSFORMATION FROM TOPOCENTRIC HORIZON TO GEOCENTRIC HORIZON

Altitude angle H and range ρ measured from the observer's position will be different than the geocentric horizon coordinates H_e and r because of a translation in the Z direction as given by the following relations:

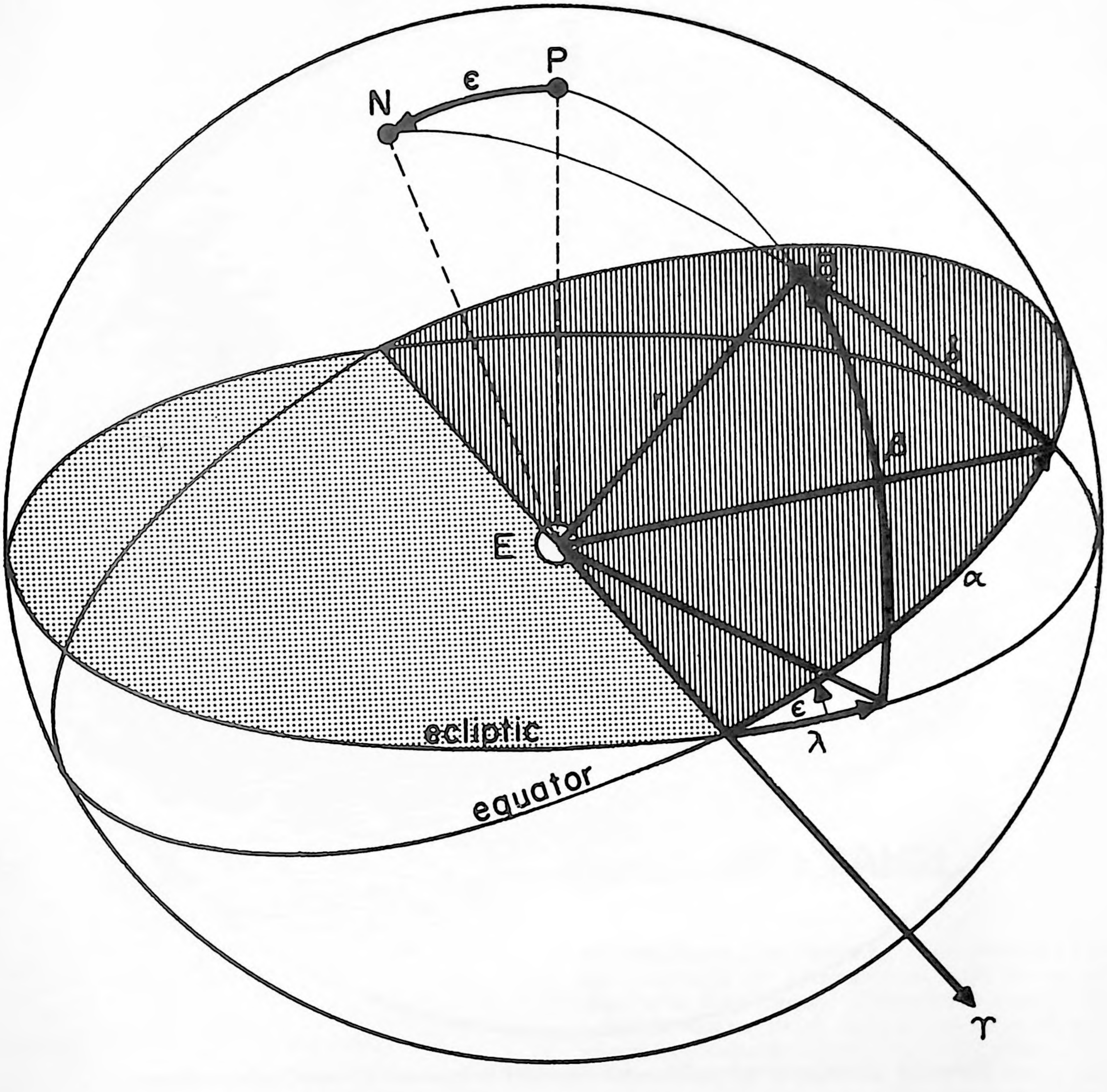

Figure 508.–Transformation from Geocentric Ecliptic to Geocentric Equatorial.

$$N_e = N = \rho \cos H \cos Az$$

$$W_e = W = \rho \cos H \sin Az$$

$$Z_e = Z + r_e = \rho \sin H + r_e$$

where r_e is the radius of the Earth at the observer's location. Then

$$r = \sqrt{N_e^2 + W_e^2 + Z_e^2}$$

$$H_e = \tan^{-1}\left[\frac{Z_e}{\sqrt{N_e^2 + W_e^2}}\right]$$

$$Az_e = Az = \tan^{-1}\frac{W_e}{N_e} .$$

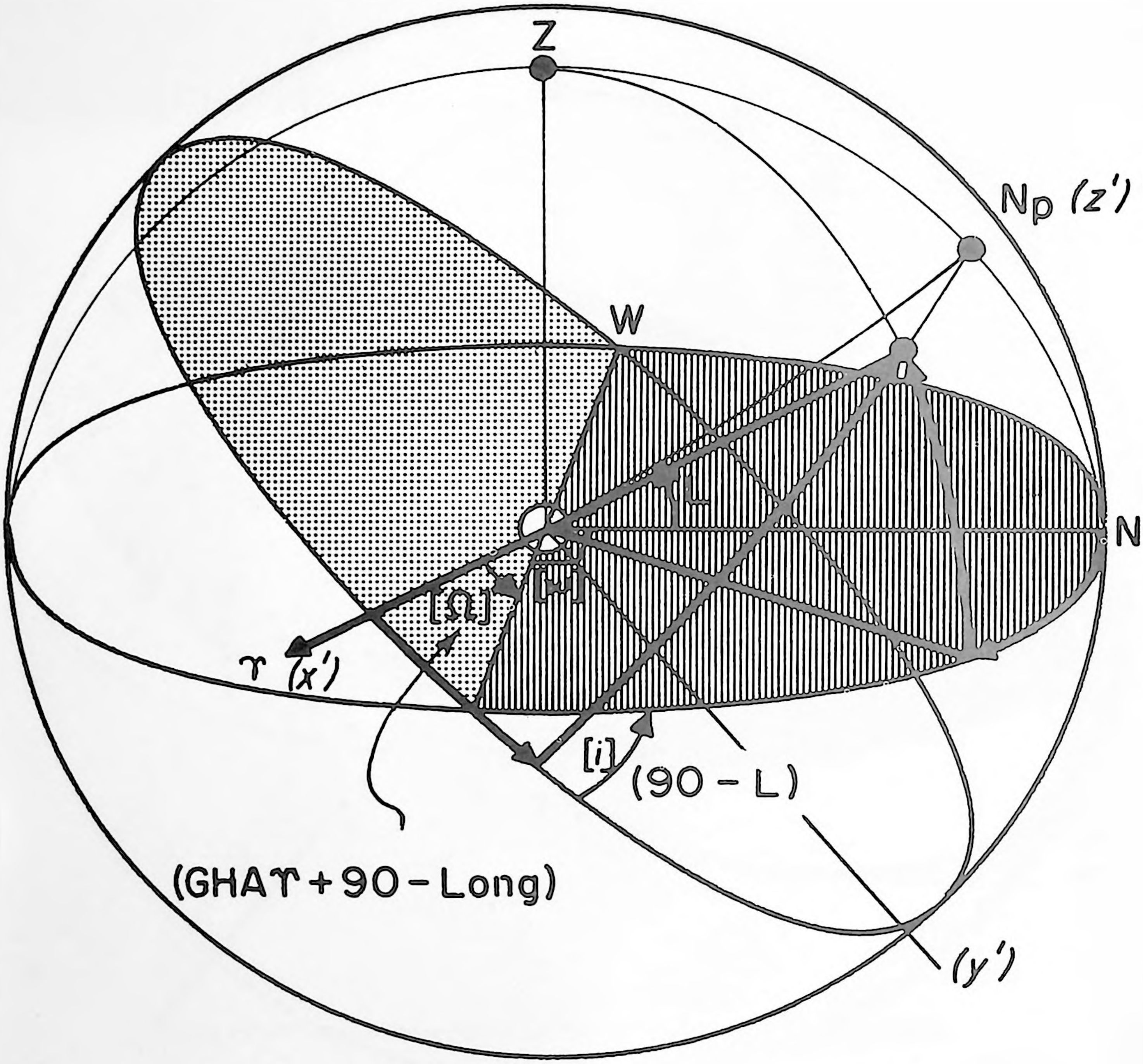

Figure 509.–Transformation from Geocentric Equatorial to Geocentric Horizon Coordinate System.

511. PARALLAX

Angle p in figures 510, 511A, and 511B is called parallax and is the difference in direction of a heavenly body as seen by an observer and as seen from the origin of a reference system (the center of the Earth).

It can also be defined as the angle at the body made by lines drawn from the body to the observer and to the center of the Earth. This last definition defines geocentric parallax. The geocentric parallax is accounted for by a transformation of topocentric coordinates into geocentric coordinates. In figure 511B the parallax p of body B is the angle OBC and

$$OBC = AOB = ZOB - ZCB.$$

From the law of sines and Δ OCB

$$\frac{OC}{CB} = \frac{\sin OBC}{\sin ZOB}$$

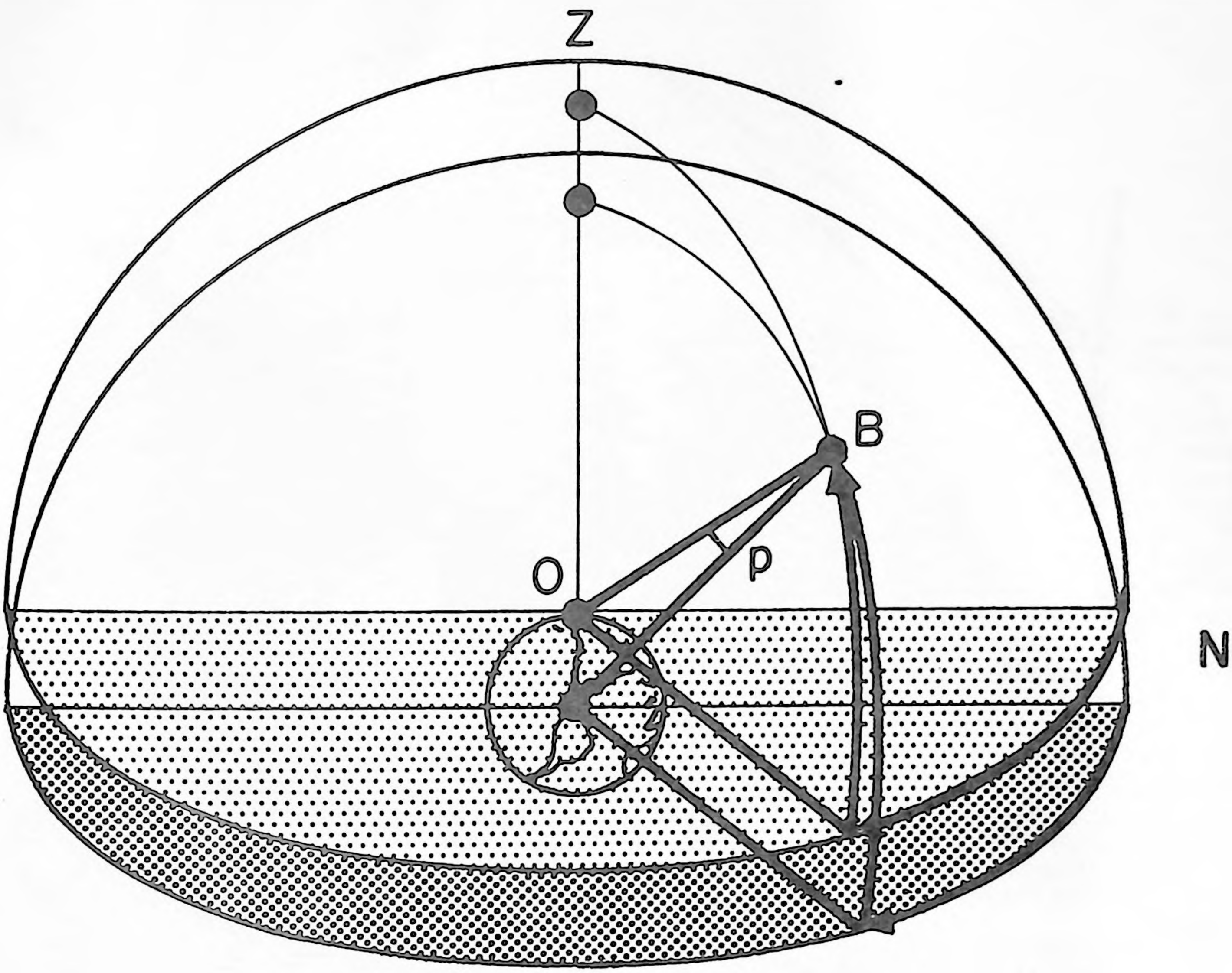

Figure 510.—Transformation from Topocentric Horizon to Geocentric Horizon Coordinate System.

or
$$\frac{r_e}{r} = \frac{\sin p}{\sin ZOB}$$

whence
$$p = \sin^{-1}\left[\frac{r_e}{r} \sin (ZOB)\right]$$

By knowing parallax p we can find the distance ρ or r and vice versa.

512. SUMMARY

These coordinate systems and the transformations from one to another have a variety of established uses, and are often necessary in the reduction of astronomical data to locate objects in the heavens. There is no single coordinate system which is best suited to all applications in space position determination.

The material is included here basically for reference purposes, and to illustrate the complexity of the problem of coordinate transformations. In the chapter on dead reckoning in space a method of predicting a body's position will be discussed which will require only one transformation, and that by graphical methods.

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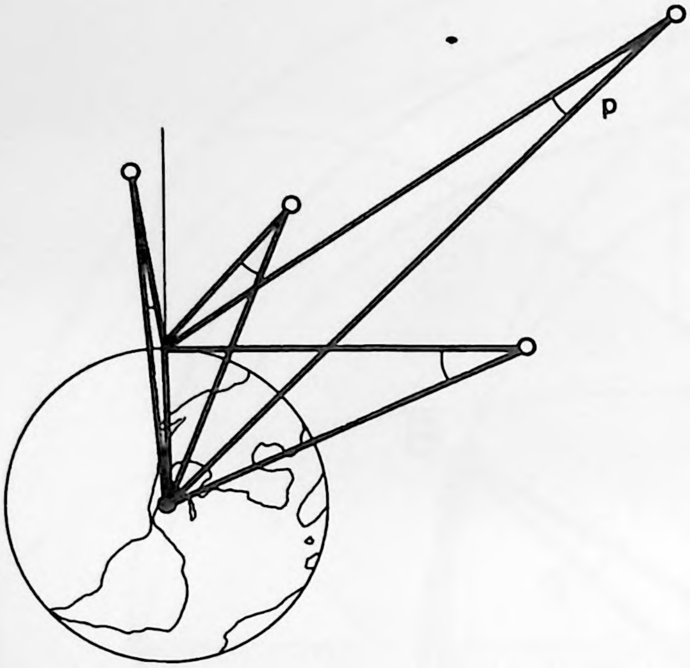

Figure 511A.–Geocentric Parallax.

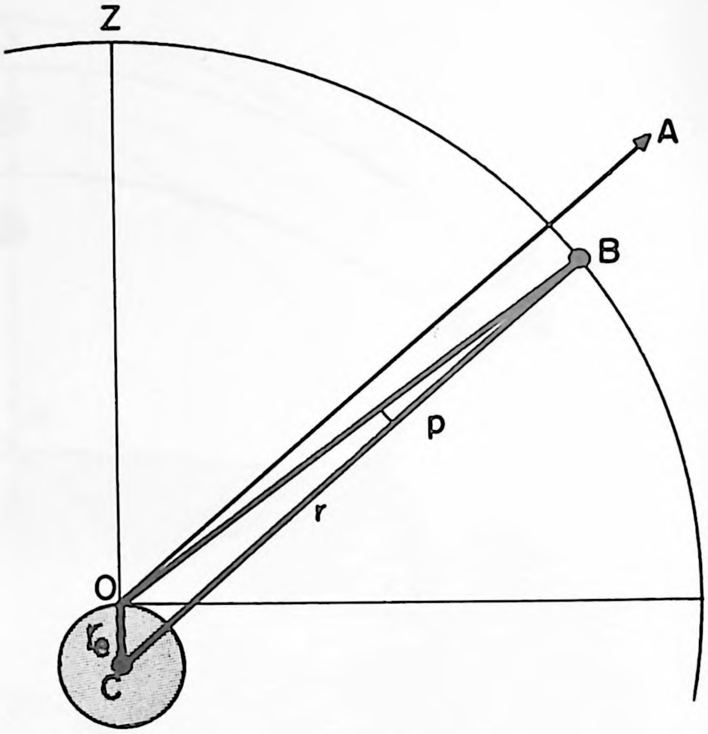

Figure 511B.–Parallax.

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2. Goldstein, Herbert. Classical Mechanics. Reading, Massachusetts, Addison-Wesley, 1959.

CHAPTER 6

EARTH-BASED TRACKING TECHNIQUES

Before going into the subject of satellite tracking by an astronaut actually in the vehicle, it is worthwhile to make a brief review of satellite tracking from the Earth. We shall endeavor to discuss the various principles involved, and then to consider the Earth systems that use these principles. One thing that we note as we progress is that these systems are tremendously complex and rely on mazes of highly specialized equipment.

601. GENERAL DISCUSSION

There are three major ways in which a satellite may be observed. An optical system may be used to detect reflected sunlight from the satellite's surfaces, or the vehicle may be equipped with a flashing light. Tracking can also be accomplished by measuring radio signals emanating from the satellite. Either an active or a passive technique is possible. In an active system, signals in the radio frequencies (RF) could be transmitted from an Earth station, bounced off the satellite, and received again on Earth. A passive tracker would receive signals generated within the vehicle itself. (We shall consider such a system to be passive, since the equipment on the Earth is nonradiating, even though an active component within the vehicle is essential.)

602. OPTICAL TRACKING

Optical observations of artificial satellites from the Earth have two distinctive features: (1) greater accuracy inherent with optical methods; and, (2) limited opportunities to make observations. It is desirable to view a satellite against a star field. It is also necessary for the satellite to be lighted by the Sun's rays. Artificial lighting for satellites would overcome this difficulty, but such methods have not yet been employed. This means that a satellite may be observed from a station only before sunrise and after sunset, and when it is at sufficient height to be outside the Earth's shadow. In general, stations have greater opportunities to make visual observations of those satellites whose orbit inclinations are numerically close to their own colatitudes.

Counterbalancing these limiting features, however, is the relative resolution of optical instruments as opposed to radiowave instruments. Resolving power is proportional to the ratio of aperture diameter to wavelength. The effective wavelength of visible light is about .55 microns, while that of a 100-megacycle radio signal is 3 meters. Were a radio instrument to have the same resolving power as an optical device with a 10 cm entrance pupil, it would need an aperture with a diameter of 340 miles. In addition an optical system can use as a reference the positions of the stars, which, according to Dr. William Markowitz of the Naval Observatory, are accurate to 0.2 seconds of arc. This eliminates instrument errors and refraction uncertainties. All serious optical observations today are based on the use of apparent sidereal coordinates.

The simplest optical sighting method is unaided visual observation. It may be used to locate a bright satellite in relation to the fixed star background to within a degree of arc. It was this method that was used as a backup for tracking the first Sputniks. Radio signals from WWV, the Bureau of Standard's time service, enabled observers to record time to within a second. A large number of such sightings resulted in refinements of the orbit determinations for the first satellites. The same principle, using more accurate predictions, permits use of large aperture telescopes and allows fixes good to at least ± 0.1 degree of arc and ± 0.1 second of time.

An entirely different approach is to use theodolites for optical tracking. A theodolite is essentially a very accurate transit. Once it is set up and tied in to the station's topocentric reference system, it can be used to record values of altitude and azimuth. The instrument must be directed so as to locate the satellite on the cross hairs. At such an instant, the time

may be recorded and angular coordinates read. Kinetheodolites permit measurements of the same sort; in addition, they photographically record the satellite image, the readings on the altitude and azimuth indicators, and the time, which is a special input to the device. Theodolite measurements must be corrected for level, collimation, azimuth, and elevation errors and for refraction by the Earth's atmosphere.

The most sophisticated method of optical tracking consists of photographing the satellite against the stars. Many types of apparatus have been developed, a large number of them for use by amateurs. The most refined of these is the Baker-Nunn camera, originally designed for tracking the first Earth satellites. The camera is of 24-inch focal length, has a 31-inch spherical mirror, a Schmidt corrector plate, and includes a precise time source. Such cameras are said to be theoretically capable of photographing satellites against a background of sixth magnitude stars, with an accuracy of 2 seconds in arc and .001 second in time. Special mounting permits the camera to either follow the stars or follow the satellite. The purpose in following the satellite is to permit integrating the light. This feature has permitted Baker-Nunn cameras to photograph Vanguard I, an object the size of a grapefruit, at slant-range distances of thousands of miles. A special barrel shutter in the focal plane causes breaks in the satellite or star trails (depending on which mode is used, star or satellite tracking) to allow time determination for one or more sets of fix points.

A series of Baker-Nunn camera sites have been set up by the Smithsonian Astrophysical Observatory (SAO). Originally under the auspices of the International Geophysical Year Committee, the Baker-Nunn stations have been taken over by the National Aeronautics and Space Administration (NASA). Baker-Nunn photographs are normally developed in the field, and field-reduced-fix data are sent to SAO headquarters for processing. The negatives are then forwarded for postanalysis.

603. DOPPLER TRACKING

Before going into the various electromagnetic systems, we shall investigate two extremely important concepts that are employed in radio tracking. The first of these is the Doppler frequency shift. The Doppler principle states that, if a frequency source is in motion relative to an observer, then the observer will receive a frequency

$$f_o = \frac{f_s}{1 - \frac{v_r}{c}}$$

where f_s is the frequency transmitted by the source, v_r is the magnitude of the radial velocity of the source with respect to the observer, and c is the speed of propagation of electromagnetic radiation in a vacuum. Since satellite positions are often measured in a coordinate frame that rotates with the Earth, and since the observer is at rest on the Earth, v_r can be simply a measure of satellite velocity in the coordinate system.

The effect of the satellite's motion is to cause a Doppler frequency shift,

$$\Delta f = f_o - f_s = \frac{v_r}{c} f_s \left(\frac{1}{1 - \frac{v_r}{c}} \right).$$

Although satellite velocities are large by terrestrial standards, they are small in comparison with the speed of light, and the term v_r/c is very much smaller than unity. Little error is incurred by making the approximation $\Delta f = (v_r/c) f_s$. The Doppler shift, although only a very small fraction of the transmitted frequency, would be in the order of 2.5 kilocycles for a typical satellite radiating at 108 megacycles. And modern radio techniques permit measurements of frequency to better than one cycle per second accuracy.

By examining a simplified example, we can get at least a qualitative picture of what actually happens during a satellite pass. Suppose a radio source radiating at frequency f_s is moving in a straight line at velocity v. Let the distance from the observer O to the source P be R, and the distance at the point of closest approach R_m. (Note that R_m is the perpendicular distance from O to the source path.) Let ζ be the angle at O

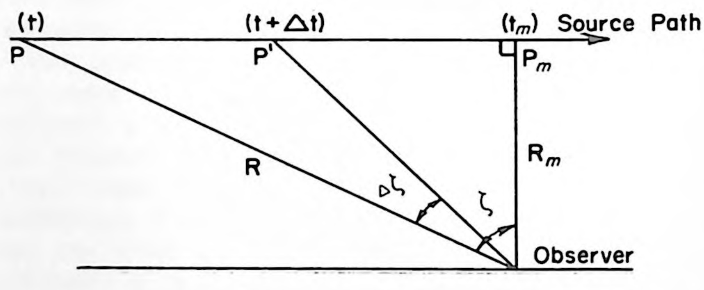

Figure 603A.–A Doppler Satellite Pass. A radiating source moves past a fixed observer at constant velocity. The instantaneous range at time t is R. The range at the point of closest approach is R_m.

between the point of closest approach and the actual position. (See figure 603A.) Note that radial velocity v_r is given by $v_r = v \sin \zeta$. As the source moves from P to P' in a time Δt, then ζ changes by an increment $\Delta\zeta$.

The Doppler shift becomes, in our notation,

$$\Delta f = \frac{f_s v \sin \zeta}{c}.$$

While the approaching satellite is very far away, ζ is nearly 90°, and $\sin \zeta \doteq 1$. The frequency received is then $f_s v/c$ above the transmitted frequency. If f_s is known, as it usually is, then v may be found. As the source (or satellite) approaches, the Doppler shift decreases, becoming zero at the point of closest approach, and then goes negative as the source moves away. A plot of Doppler shift versus time is shown in figure 603B.

Differentiating Δf with respect to time, we have

$$\frac{d(\Delta f)}{dt} = \frac{f_s v}{c} \cos \zeta \frac{d\zeta}{dt}.$$

At the inflection point (point of closest approach in this case), $d(\Delta f)/dt$ is a maximum. In addition ζ is zero at this point and $\cos \zeta = 1$. In a small neighborhood about the point of closest approach we may say that $\Delta\zeta = \omega \Delta t$, where ω is the angular frequency v/R_n. Then

$$\frac{\Delta\zeta}{\Delta t} = \omega = \frac{v}{R_n}.$$

In the limit this yields $d\zeta/dt = v/R_n$. At closest approach, then,

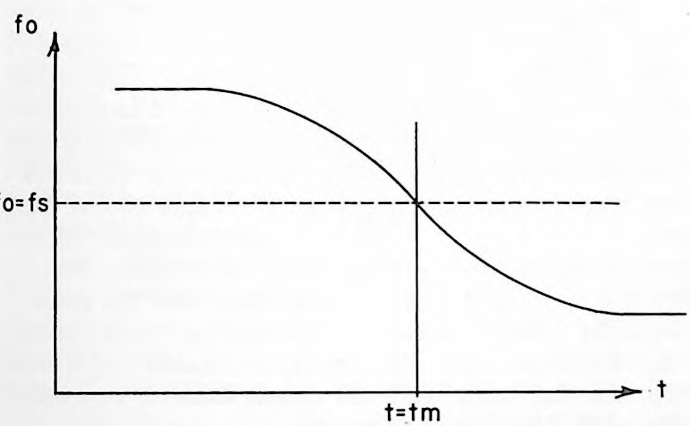

Figure 603B.–Plot of Doppler Shift Against Time. Plot of observed frequency f_o against time. The Doppler shift from the radiated frequency f_s is illustrated. The Doppler shift becomes negative as the source reaches its closest point of approach at t_m.

$$\left(\frac{d(\Delta f)}{dt}\right)_{max} \doteq \frac{f_s v}{c} \frac{v}{R_n}.$$

Rewriting, we have

$$R_n = \frac{f_s v^2}{c} \left(\frac{d(\Delta f)}{dt}\right)_{max}.$$

Thus, by recording Doppler shift as a function of time, the source velocity and the range at closest approach may be easily found.

Although the above treatment is only an approximation to an actual pass of a satellite, its results are quite close to observed values. A more rigorous investigation by Kurt Toman[1] includes the effects of the curvature of a satellite orbit, and one by Brito[2] employs the methods of spherical geometry. These refinements, of course necessary for making actual calculations from data gathered in Doppler tracking, do not invalidate the qualitative picture given by the simplified treatment. Indeed a typical plot of Doppler shift against time will closely resemble figure 603B.

Electronic computers can utilize Doppler tracking data to determine the orbital elements. One technique is "curve-fitting," i.e., using all available *a priori* information to estimate the orbit, selecting values for the orbital parameters, determining the Doppler shift curve that would result from that particular orbit, and comparing this curve to the actual curve as observed. Values of the parameters are varied until a curve is found that coincides with the observed curve.

Another method, proposed by D. R. Moorcroft[3] of the Canadian Defence Research Board, also relies on *a priori* information to estimate values of the orbital parameters. Certain of these values are inserted into the equations of motion to give more refined estimates of other parameters. An iterative process is employed, until a set of assumed parameters will remain unchanged through a complete cycle. This method, although it requires information from successive passes, is simple enough to be worked out without the use of large electronic computers.

604. INTERFEROMETER TRACKING

Another important direction finding technique utilizes the interferometer principle. If two waves of the same frequency and amplitude are combined, the resultant signal will be a function of the phase difference. If the waves are in phase, the waves will reinforce each other, the result being a wave with the same frequency

but twice the original amplitude. If the waves are 180° out of phase, the final effect is complete destructive interference and there is no output signal. Intermediate phase relationships will result in partial interference, either constructive or destructive. Waves from the same source, originally in phase, of course, may be made to produce interference patterns by sending the waves through different paths. This is the basis of the interferometer method of direction finding.

Suppose a satellite radiating in space transmits signals which are received by two adjacent antennas. Since the distance to the satellite is much greater than the antenna separations, the incoming rays are in phase and are parallel (figure 604). Unless the satellite is equidistant between the antennas, the received signals will have different path lengths. Let our basic unit of measurement be the wavelength, λ, of the received radiation. We can measure the antenna separation in terms of λ, i.e., $D = m\lambda$, where m is some number, not necessarily an integer. The difference in path length to the two antennas may also be expressed in terms of the wavelength. Let this difference be $n\lambda$. The figure shows that θ, the angle between the incoming rays and the antenna axis, is determined by the expression, $\cos \theta = n/m$. The antenna separation, and hence m, may be accurately measured by standard surveying methods. Phase detectors will measure the phase difference of the signals received by the two antennas. If the phase detector reads ϕ radians, the actual phase difference might be ϕ, or $\phi + 2\pi$, or $\phi + 4\pi$, . . . Correspondingly, n may be

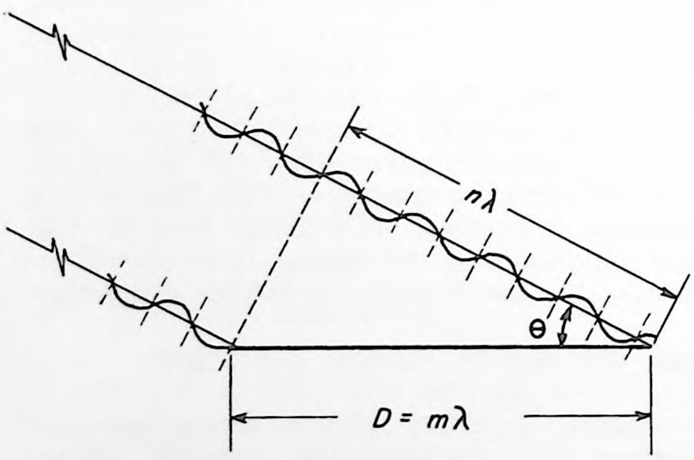

Figure 604.–The Interferometer Principle. Demonstration of the radio interferometer principle. The wavelength of the received signal is λ, which is used as the unit of measurement. The antenna axis makes an angle θ with the line to the satellite.

$$\frac{\phi}{2\pi}, \quad \frac{\phi}{2\pi} + 1, \quad \frac{\phi}{2\pi} + 2, \ldots .$$

Thus, there is a large ambiguity in the measure of n and hence of θ. If, however, several sets of antennas with varied separations are used, the ambiguity may be resolved. The normal procedure is to use an antenna pair with a short separation for a course measurement, and pairs with increasing separations for intermediate and fine readings. By comparison of the different results, the true angular coordinate may be quickly and accurately determined.

A typical tracking installation using the interferometer technique will have antennas set along two axes, which are usually perpendicular. This configuration results in minimal observational error for a given uncertainty in interferometer readings. A single interferometer observation will place the source on a cone coaxial with the two antennas. Simultaneous observations on two axes will locate the source along a line whose direction is fixed in space. Through a straightforward trigonometric transformation, the source's position can be given in terms of azimuth and elevation, as viewed by the observer.

As the satellite makes its pass over an interferometer station, phase data are recorded as a function of time. These data can be fed into a specially programmed computer, which will process the information and determine the satellite's orbit. Or observational data can be transformed into altitude and azimuth readings, both as functions of time. A careful analysis of these readings, similar to the one by Moorcroft discussed in the previous section, will result in a determination of the satellite orbit.

An interferometer system is subject to several types of errors. A level error occurs if both antennas are not on precisely the same level. In constructing the antennas, it is impossible to align the axes exactly as designed. Hence an azimuth error is introduced. Collimation errors also arise which are due to the fact that the path lengths from the antennas to the phasemeters are not always identical. Those errors are systematic, and they can be eliminated to a large extent by means of calibration. Calibrations for all interferometer systems are similar and will be described in conjunction with Minitrack tracking.

605. MINITRACK

Now that we are familiar with some radio tracking techniques, we can take a look at the various systems. The first U. S. satellite

tracking system was Minitrack, developed by the Naval Research Laboratory in connection with Project Vanguard. The heart of the system is a 108-mc oscillator in the satellite with from 10 to 50 milliwatts of output. The transmitter is minimum weight, and is powered by batteries with a two-week lifetime. On some installations, solar cells are included so that the batteries may be recharged, while on others, solar cells power the transmitters directly whenever the satellite is in sunlight. On later satellites there is a telemetric link from a control station to regulate the times of transmission, in order to keep from cluttering the 108-mc band.

Minitrack stations employ the radio interferometer method of tracking, described in the preceding section. A typical installation has antennas along north-south and east-west baselines. The eight antennas are spaced so as to form three north-south and two east-west pairs. The longest baseline in each direction is 500 feet (the wavelength is 9.1 feet). The antenna array is such that a fan-like reception pattern 100° wide and 10° deep is formed. The pattern is oriented along the north-south axis.

A complex of amplifiers, mixers, filters and detectors compare the phase characteristics of the signals from the two antennas. A 500 cps special oscillator is used as a reference, and time signals are taken from WWV. Both analog and digital displays are used; but the analog output, coupled with the WWV time signal, forms the primary display. Complete descriptions of the electronic components can be found in the literature.[5,6] The system is calibrated by having an airplane with a Minitrack transmitter fly over the station through all parts of the reception pattern. A condenser-discharge light on the plane is tracked photographically, while simultaneous radio signals are recorded at the station. Apparent optical positions are reduced and compared with the positional observations made by the system. Minitrack is designed to achieve accuracies of 1 millisecond of time and 20 seconds of arc.

The original Minitrack setup consisted of eleven stations placed strategically throughout the globe so as to provide optimum coverage. All tracking data was sent to the Naval Research Laboratory in Washington, where it was fed into a previously programmed computer. The computer calculated the orbit, and made corrections as succeeding information arrived.

NASA has since taken over the Minitrack system for their tracking purposes. Their network comprises 14 stations set up in a north-south "fence." Data is recorded and sent to NASA's Goddard Space Flight Center for computation. The transmission frequency has been changed to 135 mc, in order to get away from the upper fringes of the FM band. Minitrack, with some refined data transmission links, is still NASA's primary satellite tracking system.

606. MICROLOCK

Another passive radio system using interferometer detection is Microlock. As designed by the Jet Propulsion Laboratory, the system employs a very light weight, low-power transmitter in the satellite at the expense of heavier equipment on the ground. Microlock has capabilities both for tracking and for telemetering. Phase-coherent reference signals are relayed to the interferometer receiver through a phase-locked receiver, which is the heart of the telemetering equipment.[6-8] Thus, correlation detection of the signal is possible.

The transmitter weighs only two pounds with batteries and produces 3 milliwatts. It is designed to transmit at 108 mc for three months. Like Minitrack, time and frequency standards are received from WWV.

The principal advantage of the system is that lower power signals can be received. The use of a phase-locked receiver enables the system to track the signal effectively even when noise levels are comparable to or even greater than signal levels. Other difficulties are encountered, however, in the acquisition of these low-power signals. The Microlock stations are mounted in mobile trailers and can be shifted to provide optimum tracking in various situations. Microlock, of which 8 to 10 units have been built, has not become a prime tracking system.

607. DOPLOC

An outgrowth of the telemetering phase of the Microlock program is DOPLOC, an active tracking system. DOPLOC, which stands for Doppler Phase Lock, determines satellite orbits by measuring the Doppler shift. It receives very low-power signals near the 108-mc band. A phase-locked receiver permits reception on a very narrow band filter, normally operated around 5 cps, and hence pickup of weak signals. The receiver still has the capability to follow the change in frequency as the satellite passes. (This change can be as much as 6 kc for a given satellite pass.) The phase-lock apparatus compares the received signal to a reference signal

from a crystal oscillator, and then produces an error signal. The error signal drives a servomechanism which tunes the low-pass filter in such a way that the error becomes zero. Thus the system has an automatic tuning feature.

The antenna configuration comprises three separate systems, each of which forms a fan beam. Passes through the fans permit determination of segments of the Doppler shift curve. Both analog and digital records are made, allowing Doppler shift to be measured with errors of only a fraction of a cycle per second.

Data is then sent to an electronic computer for processing. The computer mathematically derives Doppler curves from the six orbital elements, coupled with parameters representing refractive corrections and electron densities in the ionosphere. The parameters are varied until a theoretical curve is found which fits the recorded data. This technique enables determination of an orbit by data gathered on a single pass of a satellite. Results obtained in this manner have compared very favorably with elements determined by other systems using multiple-pass observations.[8]

A number of DOPLOC installations have been completed by the U. S. Army. The Applied Physics Laboratory of Johns Hopkins University has also set up similar stations for tracking radiating satellites, presumably for use in conjunction with the Navy's navigational satellite program, Project Transit. Although DOPLOC was originally supported by the Department of Defense's Advanced Research Projects Agency, it has since been dropped and has not become a primary tracking system.

608. RADAR

The most obvious of the active tracking systems is radar, which has been used so effectively to detect and track surface ships and aircraft. Radar has two very desirable features: a range-finding ability coupled with a direction-finding capability. But its basic design also gives it inherent disadvantages for use as a satellite tracker. A radar transmitter must send out sharply pulsed signals, rather than a continuous wave, in order to find range to the satellite. Although interferometer and Doppler techniques are possible, directional antennas are normally used to receive reflected radar signals.

The adoption of the directional antenna results in a low degree of accuracy, for if the beam width is to be large enough to allow tracking with a reasonable acquisition time, there will be a substantial error in direction finding. The signal pulsing feature results in the need for very high power signals, or very large trainable antennas, or both.

Despite these limitations, radar has been used to some extent in satellite tracking, and research efforts promise improvements that will increase its efficiency. The Army has built several large radars, including a 60-foot parabolic dish, that will be used to track the Tiros weather satellites. The newer of these radars include an automatic tracking feature. Other radar tracking installations have been developed for use with specific satellite programs. Radars of the Ballistic Missile Early Warning System (BMEWS) have also contributed information to the satellite tracking and orbit computation centers. But another system has overshadowed radar.

609. SPACE SURVEILLANCE

The responsibility for the detection and tracking of all enemy satellites has been given to the Department of Defense. Since new satellites can be detected only if all others are known and tracked, all satellites must be tracked, including those not radiating radio signals. The result has been the development of the Navy's Space Surveillance System under the sponsorship of the Advanced Research Projects Agency.

Space Surveillance is an active tracking system employing interferometer methods. High power radio signals are generated by a transmitting station, reflected by a satellite as it passes, and received by radio interferometer receivers. The basic principle is illustrated by figure 609A.

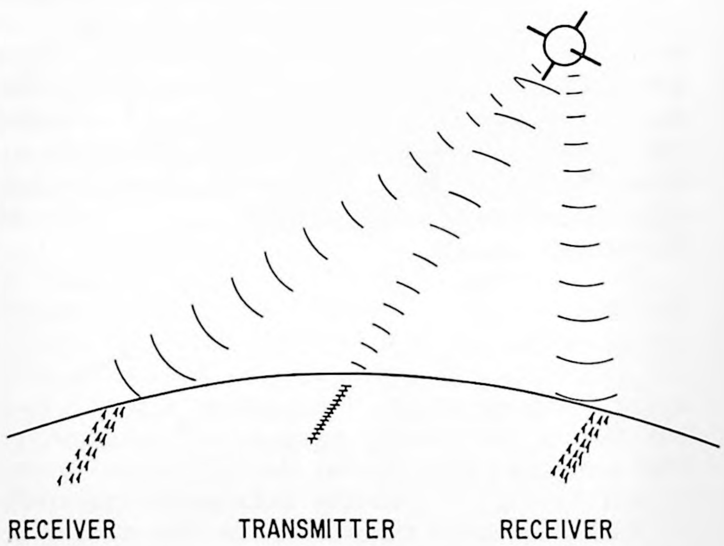

Figure 609A.–The Space Surveillance Principle. The principle of the Space Surveillance System is demonstrated.

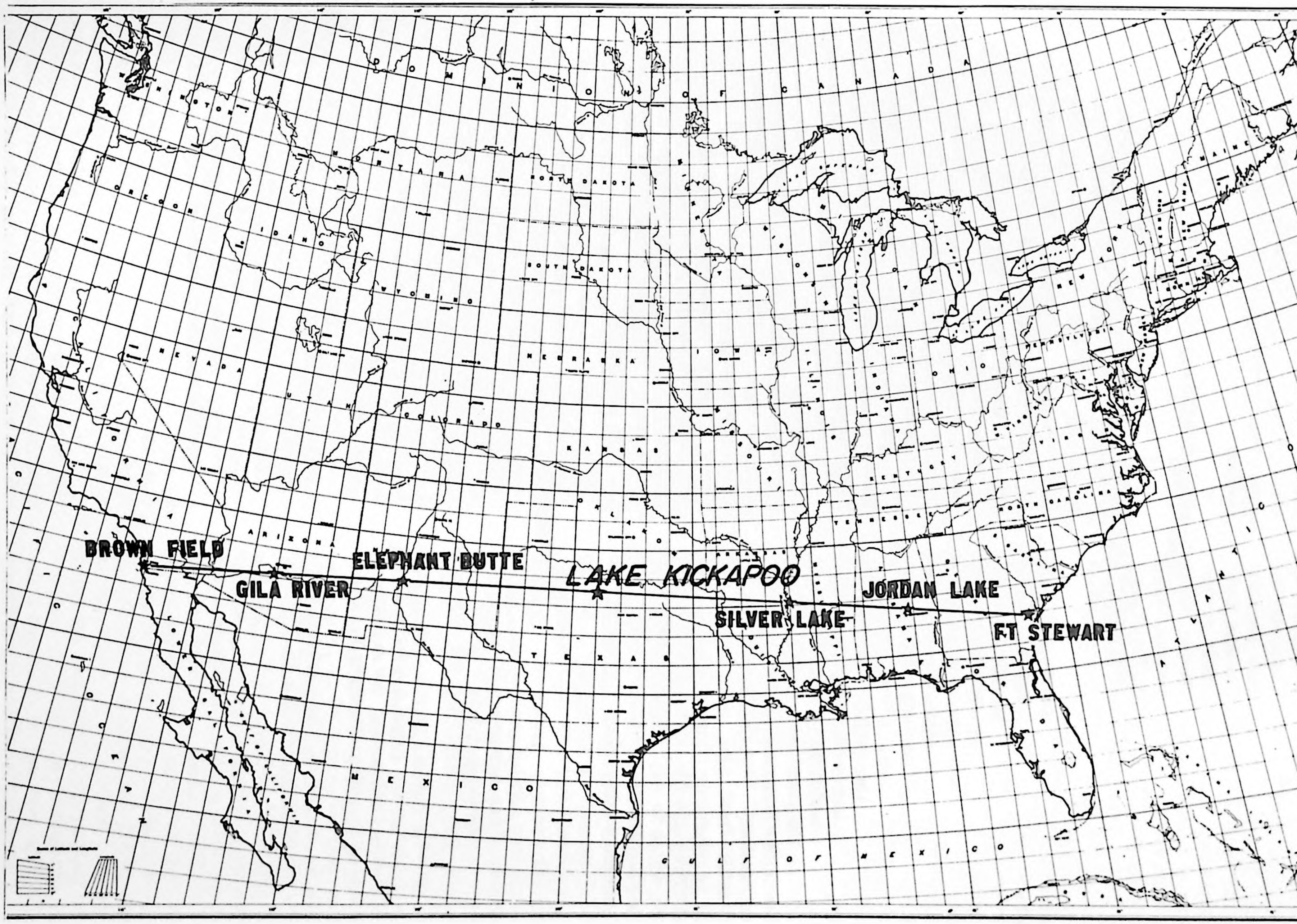

Figure 609B.—The Space Surveillance Network. The network of Space Surveillance installations. Transmitters are located at Gila River, Lake Kickapoo, and Jordan Lake. The remaining four sites are receiving stations.

The transmitting and receiving stations of Space Surveillance form a fence across the United States. The present network includes transmitters at Gila River and Jordan Lake (shown in figure 609B), and an extremely powerful transmitter at Lake Kickapoo, Texas, which has recently been added. These stations, along with the receiving stations at Brown Field, Elephant Butte, Silver Lake, and Fort Stewart, lie on a great circle. Figure 609B, a gnomonic projection, illustrates this.

The transmitters set up a narrow fan beam which is coplanar with the great circle. As the satellite moves through this narrow fence, it reflects the strong radio waves, which are then picked up by the interferometer antennas. The receiving stations are quite typical, and closely resemble the Minitrack receiving system already described.

Information from the phase detectors is placed on direct reading recorders, and is also relayed to Space Surveillance Headquarters. Here data is fed into the large NORC (Naval Ordnance Research Computer), and orbital elements are determined. The NORC memory retains information from past observations, so the computer is constantly improving the orbit. It is planned to replace the NORC computer with an IBM 7090, which is well adapted to the type of calculations involved in the Space Surveillance System. There are also plans to extend the scope of the system by establishing additional stations and by incorporating a ranging feature with the interferometer direction-finding capabilities.

610. TRACKING CENTERS

The Department of Defense has assigned satellite detection and tracking responsibilities to NORAD, the North American Air Defense Command. NORAD has organized the Space Detection and Tracking System (SPADATS) at its headquarters in Colorado Springs. SPADATS gathers data from numerous sources to carry out its mission. The Navy Space Surveillance System falls under its operational control and supplies it with information. Data is received from many other sources, some of which have been mentioned above. SPADATS gathers and interprets all tracking data as the nation's primary military tracking control group.

We have briefly reviewed the principal tracking methods, as well as some of the systems that have been developed. A most easily discernible fact is the great complexity of the equipment involved. In addition to the complicated tracking apparatus, sophisticated electronic data processing systems are necessary to convert the raw observations into usable information. This, then, sets the stage for the material to follow.

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CHAPTER 7

SATELLITE-BASED TRACKING TECHNIQUES

"The radio beam's best effort cannot, to me compare with the stark independence and simplicity of the star navigator's apparatus and methods. He depends on no fallible mass of 'electrickery' and its human crew. In 'this majestical roof, fretted with golden fire' (Shakespeare haunts this page today!) there are a million leading lamps trimmed by a steadier than mortal hand and located in eternity with divine exactitude." — an unknown British author.

701. SIMPLIFICATION OF TRACKING PROBLEM

It becomes apparent from chapter 6 that Earth-bound observers are handicapped by a number of disadvantages, and that Earth-based tracking systems can be enormously complex in their makeup. At an observation station on the Earth we need to know our position accurately, and have elaborate observing, computing, and communicating equipment. With optical tracking equipment we would suffer from night-day phenomena, and would be able to see Earth satellites only a small fraction of the time. We would also be subjected to the variable weather conditions of rain, cloud cover, fog, etc. With both optical and electronic tracking systems we must correct our observations for atmospheric refraction, and, because we are not at the center of the Earth, for parallax also.

On the other hand, the Earth-bound observer works under normal and familiar conditions, and can have almost unlimited assistance in his measurements. For example, two or more observers at different stations may combine the results of their direction measurements at a given time, and triangulate for the range of some satellite. Electronic computers provide answers rapidly, thus saving the observer from tedious numerical work and allowing him to proceed at a leisurely pace.

While the space navigator cannot have unlimited assistance, and works under conditions which are hardly normal and familiar by our standards, he does have a number of distinct advantages when tracking his satellite. Since he is outside the atmosphere for all practical purposes, he is free of night-day phenomena, variable weather and refraction conditions, and parallax. The faint light of the stars which reaches him is not scattered, so he can see many stars as well as the Earth quite distinctly, except in the region very close to the Sun. Most important, however, is the fact that the astronaut can easily accomplish a feat which is impossible for the ground observers—he can look directly down his vertical and optically locate his own geographic position.

702. IMPORTANCE OF ASTRONAUT'S VERTICAL

The significance of the observer's vertical was first suggested in chapter 1. It is a most convenient fact that the astronaut, unlike the mariner or aviator, can see the center of the Earth, and thus establish a reference for measurements which is independent of the force of gravity. If the astronaut looks in the exact direction of the Earth's center, his line of sight determines his vertical and passes through his zenith and his nadir. The point where this line intersects the near side of the Earth's surface is his geographic position. The astronaut would, of course, be in space, yet this position is just as positively fixed as if he were on the surface. His important vertical line is illustrated in figure 702. The specification of this line of position in space, by giving the two coordinates of his geographical position, is a most logical and obvious basis for fixing positions in cislunar space navigation. Range of the space vehicle above its geographic position will complete a unique three-dimensional fix in space relative to the Earth. This method of position finding, using for a basis what will appear as the largest chunk of real estate in the heavens, is made even more desirable by the ease with which the three coordinates may be determined.

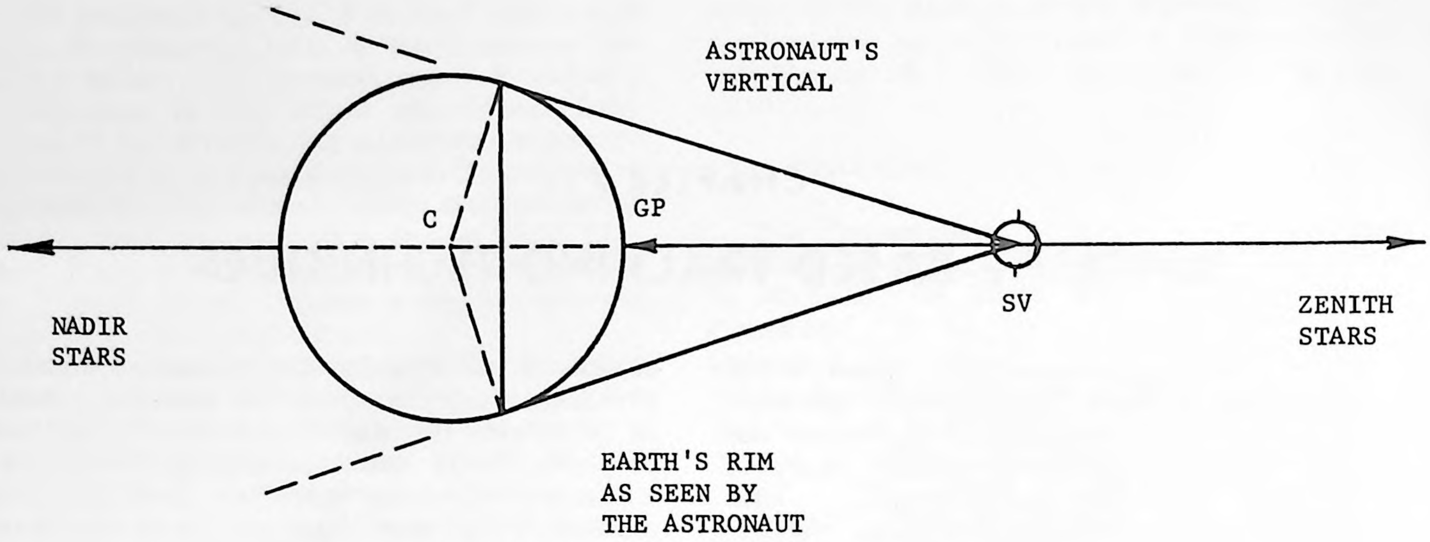

Figure 702.–Significance of Astronaut's Vertical.

703. DETERMINING CELESTIAL COORDINATES

Since we propose to use the Earth as the basis for position finding, we naturally should use one of the geocentric coordinate systems. Near the surface of the Earth, especially during the launch or reentry phases of a trip, it will be important to know the geographic position of the space vehicle in terms of latitude and longitude. But at other times we will be less interested in terrestrial coordinates, since the spin of the Earth hardly affects the motion of the vehicle once out of the atmosphere. The orbital plane of an Earth satellite will remain almost fixed in space, relative to the direction of the fixed stars. Therefore we shall find it often more convenient to use the geocentric equatorial coordinates, declination and sidereal hour angle (SHA). The conversion back to terrestrial latitude and longitude may be effected quite easily, as we shall see in the following section.

When the astronaut looks back at the Earth from his space vehicle, it will appear as a distinct and rather large disk in a very bright star field—somewhat the same as the Moon appears to Earth-bound observers on a very clear night. If he optically locates the center of this disk and projects it onto the star field behind the Earth, he will have determined his own nadir. This point in the star field must then be identified by its declination and SHA. The astronaut's line of sight reversed determines his zenith, with which we identify his geographical position in this coordinate system. Once the coordinates of the nadir are found, those of the zenith follow immediately by simply changing the name of the declination and either adding 180° to or subtracting 180° from the SHA.

Thus we may use either of two approaches in determining the sidereal coordinates of an orbiting space vehicle. We can project the center of the Earth's disk back onto the star field, locate our nadir, and indirectly find the geographic position or zenith of the vehicle, as illustrated in figure 703A. Or we may use the center of the Earth's disk to establish our vertical line, and project this line out to the zenith directly. One device using this approach, the so-called bazooka telescope, is shown in figure 703B. There seems to be no distinct advantage in looking directly at the zenith star field, and indeed it complicates the design of optical equipment. Therefore the former method is emphasized in sections 706 and 707, which describe specific methods and devices for effecting measurements of these coordinates.

704. CONVERSION TO TERRESTRIAL COORDINATES

It is a simple task to convert the celestial coordinates of declination and SHA to the terrestrial coordinates of latitude and longitude, provided that sidereal time, or the Greenwich hour angle of Aries, is known. Latitude and declination are both angles measured at the center of the Earth from the equator toward either pole, although they may be considered as arc distances on different spheres. As far as the astronaut will be concerned, the terms can be interchanged, and the conversion is done.

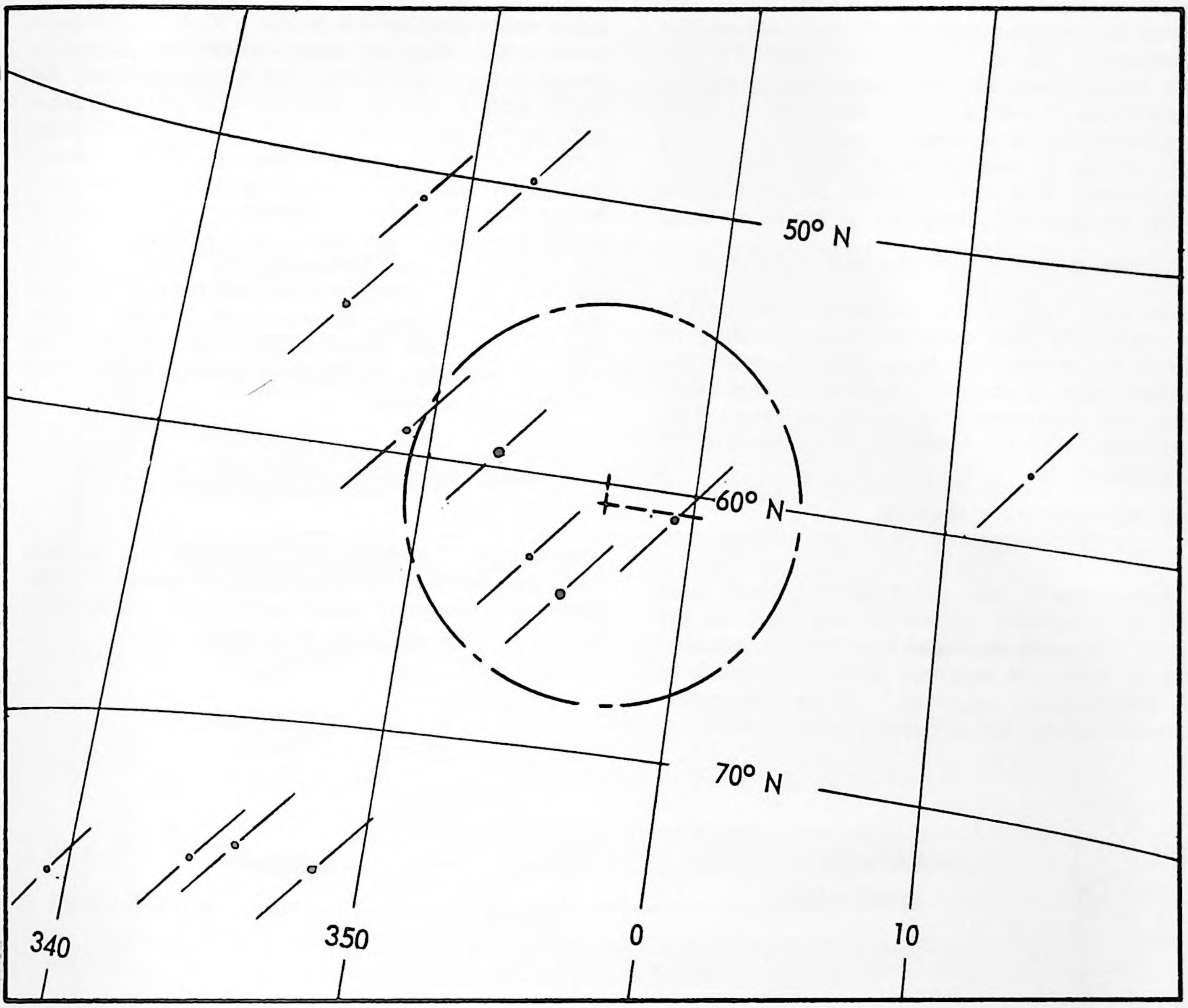

Figure 703A.—Earth's disk projected against star field, or Nadir method.

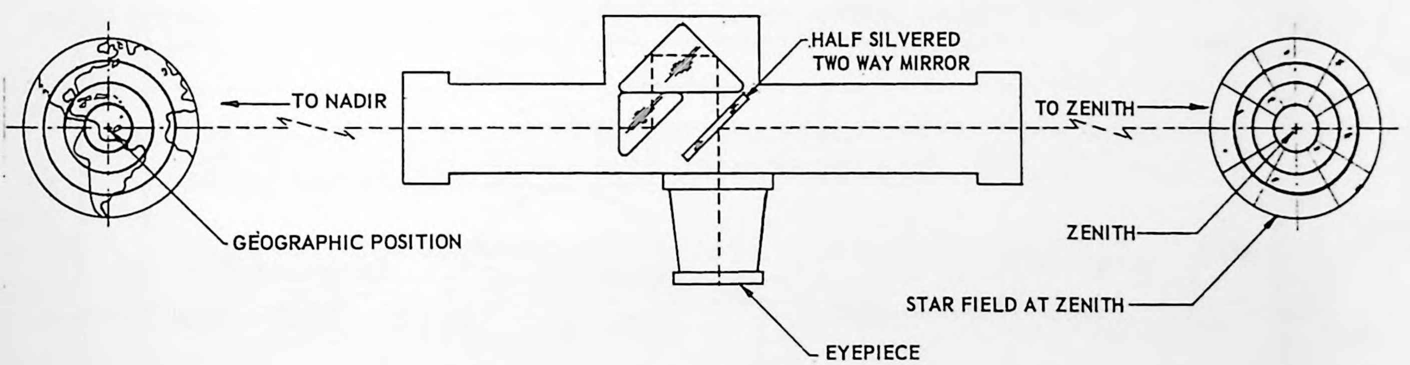

Figure 703B.—Bazooka telescope for viewing Zenith stars.

For the conversion to longitude, suppose the Greenwich hour angle of Aries (GHA Υ) has been determined for the instant in question, either from a sidereal clock directly, or from an almanac or table used in conjunction with a clock giving Greenwich Mean Time. The relation between SHA and longitude is well known among navigators to be given by the expression

$$\text{SHA}\,\bigstar + \text{GHA}\,\Upsilon = \text{GHA}\,\bigstar = \lambda(W),$$

where $\lambda(W)$ represents longitude measured to the west. If this quantity exceeds 180°, it should be subtracted from 360° to yield the correct value of eastern longitude. These relations are explained graphically in figure 704. This completes the conversion to terrestrial coordinates.

705. METHODS OF RANGE DETERMINATION

Astronomers and geophysicists have been able to determine accurately the size of the Earth. Its mean radius is known to be approximately 3,440.189 nautical miles according to the International spheroid.[1] Using this established baseline, the astronaut may observe the angle subtended by the Earth's disk at his space vehicle and find his range above the Earth by one of several methods. He might measure the angle with a marine sextant and enter a table such as the one in figure 705A to determine his range. Or he might be equipped with a sort of stadimeter designed for this baseline from which he could read off the range directly. Finally he could measure the angle subtended and solve the right triangle formed by the Earth's radius and his lines of sight to the Earth's center and rim. This triangle, which is illustrated in figure 705B, is the basis of all such stadimetric ranging methods. It follows from the geometry of the problem that

$$R/D = \sin\frac{1}{2}A \quad \text{or} \quad D = R\csc\frac{1}{2}A,$$

where A is the angle subtended by the Earth's disk, and D is the distance of the space vehicle from the center of the Earth. Hence, the altitude above the surface is given by

$$a = D - R = R\left(\csc\frac{1}{2}A - 1\right).$$

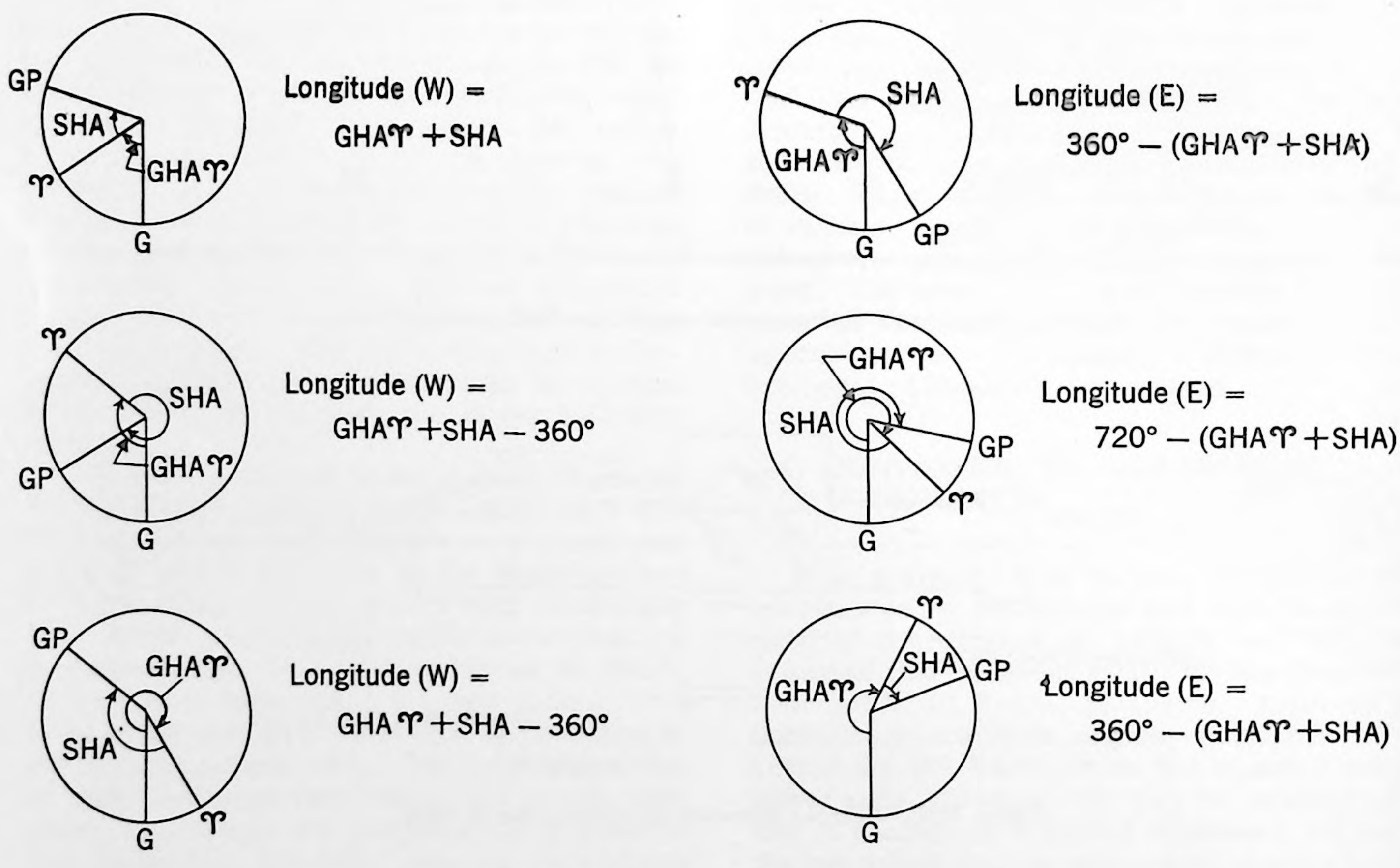

Figure 704.–Relations between SHA, GHA, and longitude.

A deg.	R N.M.	E N.M.	A deg.	R N.M.	E N.M.	A deg.	R N.M.	E N.M.	A deg.	R N.M.	E N.M.	A deg.	R N.M.	E N.M.
1.50	259378.45	292.00	4.20	90441.94	37.24	9.40	38544.82	7.43	33.00	8672.50	.59	83.00	1751.61	.09
1.55	250900.97	273.47	4.30	88259.62	35.53	9.50	38103.87	7.27	33.50	8496.78	.58	84.00	1701.09	.08
1.60	242953.24	256.64	4.40	86176.52	33.93	9.60	37672.12	7.12	34.00	8326.30	.56	85.00	1651.93	.08
1.65	235487.31	241.32	4.50	84186.02	32.44	9.70	37249.27	6.97	34.50	8160.85	.54	86.00	1604.09	.08
1.70	228460.47	227.34	4.60	82282.09	31.04	9.80	36835.07	6.83	35.00	8000.19	.53	87.00	1557.51	.08
1.75	221835.27	214.53	4.70	80459.21	29.73	9.90	36429.25	6.70	36.00	7692.50	.50	88.00	1512.16	.07
1.80	215578.05	202.78	4.80	78712.30	28.51	10.00	36031.55	6.56	37.00	7401.72	.47	89.00	1467.99	.07
1.85	209659.17	191.96	4.90	77036.71	27.36	10.50	34156.84	5.95	38.00	7126.54	.45	90.00	1424.97	.07
1.90	204051.82	181.99	5.00	75428.16	26.27	11.00	32452.78	5.42	39.00	6865.74	.42	91.00	1383.07	.07
1.95	198731.97	172.78	5.10	73882.71	25.25	11.50	30897.13	4.96	40.00	6618.25	.40	92.00	1342.24	.07
2.00	193678.19	164.25	5.20	72396.73	24.29	12.00	29471.32	4.55	41.00	6383.10	.38	93.00	1302.45	.07
2.05	188870.88	156.33	5.30	70966.83	23.38	12.50	28159.77	4.20	42.00	6159.41	.36	94.00	1263.68	.06
2.10	184292.56	148.98	5.40	69589.92	22.52	13.00	26949.31	3.88	43.00	5946.38	.35	95.00	1225.88	.06
2.15	179927.15	142.13	5.50	68263.09	21.71	13.50	25828.70	3.60	44.00	5743.28	.33	96.00	1189.04	.06
2.20	175760.23	135.74	5.60	66983.67	20.94	14.00	24788.31	3.34	45.00	5549.46	.32	97.00	1153.13	.06
2.25	171778.46	129.77	5.70	65749.15	20.21	14.50	23819.85	3.12	46.00	5364.30	.30	98.00	1118.11	.06
2.30	167969.88	124.19	5.80	64557.23	19.52	15.00	22916.12	2.91	47.00	5187.26	.29	99.00	1083.96	.06
2.35	164323.37	118.96	5.90	63405.72	18.87	15.50	22070.86	2.73	48.00	5017.84	.28	100.00	1050.66	.05
2.40	160828.77	114.06	6.00	62292.61	18.24	16.00	21278.59	2.56	49.00	4855.55	.26	101.00	1018.18	.05
2.45	157476.85	109.45	6.10	61216.02	17.65	16.50	20534.49	2.40	50.00	4699.99	.25	102.00	986.51	.05
2.50	154258.98	105.12	6.20	60174.17	17.08	17.00	19834.30	2.27	51.00	4550.75	.24	103.00	955.61	.05
2.55	151167.35	101.03	6.30	59165.41	16.55	17.50	19174.28	2.14	52.00	4407.47	.23	104.00	925.47	.05
2.60	148194.61	97.19	6.40	58188.19	16.03	18.00	18551.06	2.02	53.00	4269.82	.22	105.00	896.07	.05
2.65	145334.09	93.55	6.50	57241.06	15.54	18.50	17961.67	1.91	54.00	4137.48	.22	106.00	867.39	.05
2.70	142579.48	90.12	6.60	56322.64	15.07	19.00	17403.43	1.81	55.00	4010.16	.21	107.00	839.41	.05
2.75	139925.09	86.87	6.70	55431.65	14.63	19.50	16873.95	1.72	56.00	3887.60	.20	108.00	812.12	.04
2.80	137365.50	83.80	6.80	54566.89	14.20	20.00	16371.07	1.63	57.00	3769.55	.19	109.00	785.49	.04
2.85	134895.71	80.88	6.90	53727.19	13.79	20.50	15892.85	1.55	58.00	3655.77	.19	110.00	759.51	.04
2.90	132511.11	78.12	7.00	52911.51	13.40	21.00	15437.52	1.48	59.00	3546.05	.18	111.00	734.16	.04
2.95	130207.34	75.49	7.10	52118.82	13.03	21.50	15003.49	1.41	60.00	3440.19	.17	112.00	709.43	.04
3.00	127980.38	72.99	7.20	51348.16	12.67	22.00	14589.30	1.35	61.00	3338.00	.17	113.00	685.30	.04
3.05	125826.43	70.62	7.30	50598.63	12.32	22.50	14193.64	1.29	62.00	3239.30	.16	114.00	661.77	.04
3.10	123741.99	68.36	7.40	49869.37	11.99	23.00	13815.29	1.23	63.00	3143.92	.16	115.00	638.81	.04
3.15	121723.71	66.21	7.50	49159.58	11.67	23.50	13453.15	1.18	64.00	3051.72	.15	116.00	616.41	.04
3.20	119768.53	64.15	7.60	48468.47	11.37	24.00	13106.21	1.13	65.00	2962.55	.15	117.00	594.56	.04
3.25	117873.52	62.20	7.70	47795.32	11.07	24.50	12773.53	1.09	66.00	2876.27	.14	118.00	573.25	.04
3.30	116035.91	60.32	7.80	47139.45	10.79	25.00	12454.26	1.04	67.00	2792.75	.14	119.00	552.47	.03
3.35	114253.20	58.54	7.90	46500.20	10.52	25.50	12147.62	1.00	68.00	2711.87	.13	120.00	532.20	.03
3.40	112522.90	56.83	8.00	45876.94	10.26	26.00	11852.87	.96	69.00	2633.52	.13	121.00	512.44	.03
3.45	110842.78	55.19	8.10	45269.08	10.01	26.50	11569.34	.93	70.00	2557.60	.12	122.00	493.16	.03
3.50	109210.65	53.63	8.20	44676.06	9.76	27.00	11296.40	.89	71.00	2483.99	.12	123.00	474.38	.03
3.55	107624.52	52.13	8.30	44097.34	9.53	27.50	11033.49	.86	72.00	2412.61	.12	124.00	456.07	.03
3.60	106082.45	50.69	8.40	43532.42	9.30	28.00	10780.06	.83	73.00	2343.36	.11	125.00	438.22	.03
3.65	104582.64	49.31	8.50	42980.80	9.09	28.50	10535.61	.80	74.00	2276.17	.11	126.00	420.82	.03
3.70	103123.36	47.98	8.60	42442.01	8.88	29.00	10299.68	.77	75.00	2210.94	.11	127.00	403.88	.03
3.75	101703.02	46.71	8.70	41915.63	8.67	29.50	10071.84	.75	76.00	2147.60	.10	128.00	387.37	.03
3.80	100320.06	45.49	8.80	41401.22	8.48	30.00	9851.68	.72	77.00	2086.09	.10	129.00	371.29	.03
3.85	98973.01	44.32	8.90	40898.37	8.29	30.50	9638.82	.70	78.00	2026.33	.10	130.00	355.64	.03
3.90	97660.53	43.19	9.00	40406.72	8.10	31.00	9432.92	.68	79.00	1968.25	.10	131.00	340.40	.03
3.95	96381.25	42.10	9.10	39925.88	7.93	31.50	9233.64	.65	80.00	1911.79	.09	132.00	325.57	.02
4.00	95133.99	41.06	9.20	39455.51	7.75	32.00	9040.66	.63	81.00	1856.91	.09	133.00	311.13	.02
4.10	92730.73	39.08	9.30	38995.27	7.59	32.50	8853.71	.61	82.00	1803.53	.09	134.00	297.10	.02

Figure 705A.–Stadimetric Range/Error Table.

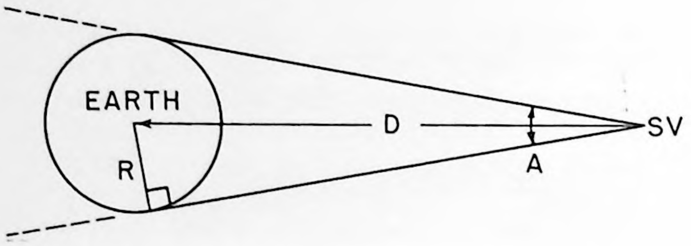

Figure 705B.–Geometry of Stadimetric Range Determination.

The accuracy of such range finding depends upon the accuracy with which the subtended angle can be measured, as well as the distance from the baseline. If we assume that the baseline is a precisely determined constant, then the variation in distance as a function of variation in the measured angle A is given by the expression

$$\delta D = R \cdot \delta \csc \frac{1}{2} A = - R \cdot \csc \frac{1}{2} A \cdot \cot \frac{1}{2} A \cdot \frac{1}{2} \delta A$$

where the angle error δA is given in radians. The resulting range error δD is given in the same units as the Earth radius R. Assuming that the subtended angle is measured with a good marine sextant by a skilled navigator, the average error in the measured angle would be about 0.1 minute. If the space vehicle were at a distance of 1,000 miles above the earth, this would result in a range uncertainty of 0.053 miles. But at 100,000 miles from the Earth the same angular error would cause a range error of approximately 45 miles. The third column of the table in figure 705A gives the uncertainty in range for an angle error of 0.1 minute of arc, or 0.00166667°. It should be noted that for observations near the Earth the principal source of error will not be in the angle-measuring device itself, but in the uncertainty of the baseline, the varying refraction conditions of the atmosphere, and false horizons caused by cloud cover and phase changes of the Earth. It obviously becomes impractical to use this method of altitude determination below about 500 miles when the subtended angle approaches 120°.

An alternative closely related to the stadimetric method is that of measuring the curvature of a portion of the Earth's rim. The analysis is similar to that above, but the observer need only see a small part of the Earth's rim. In one configuration using this method, the observer views an image of the Earth projected through a template on which are drawn a number of eccentric circles, each labeled for a particular altitude. By pointing the device he matches the edge of the image with one of the circles and estimates the altitude. This design[2] is described further in section 707.

Other methods of range determination fall in the broad category of electronic devices. Microwave radar has the advantage of giving very accurate range measurements, and its error is fairly independent of the range itself. However, it becomes impractical over extreme distances, such as those involved in space navigation, because of the large beam power required. A new device known as the optical maser,[3-5] or laser, has recently been developed which overcomes this power problem by confining its electromagnetic radiation to an extremely narrow beamwidth (of the order of one microradian). Because the wavelength used is many times smaller than that of microwave radars, only very small reflectors will be required. Some preliminary predictions[6] indicate that in the next few years less than 100 watts of average beam power will permit measurement of distances in space around 160,000 km with an accuracy of one part in 100,000.

Further discussion of these electronic ranging devices is considered to be beyond the scope of this Handbook. However, the reader may consult some of the references listed at the end of this chapter.

706. GEOGRAPHIC POSITION DETERMINATION WITH A SEXTANT

The ordinary marine sextant may be used to determine the astronaut's geographic position, in addition to the stadimetric range. Suppose the sextant were so constructed that faint concentric circles centered on the telescope axis were visible in the optical field. Then by direct vision the astronaut could center the instrument on the Earth's disk and measure angular distances to selected nearby stars by means of these circles, though a relatively crude measurement would result. However, the fullest use of the instrument's capability could be made by viewing the selected star directly and moving the arm until the Earth's rim just appears to touch the star. The marine navigator uses this same technique to measure the altitude above the horizon of a faint star. For the astronaut, the method expands his possible field of measurements from a few degrees to about 120°, in addition to giving much greater accuracy.

To find his geographic position by the method outlined in the preceding sections, the astronaut must ultimately measure the angular distances between the Earth's rim and two or three known

stars. Since he must measure the apparent angular diameter of the Earth's disk for stadimetric range determination anyway, he can apply half this subtended angle to the measured star altitude and accurately determine the angular distances of these stars from the Earth's center. Assuming these required angles have been found with a sextant, or similar device, the astronaut can then determine the coordinates of his nadir by one of two general methods.

The first and simplest approach for finding these coordinates is to draw circles on a globe[7] which has the positions of navigational stars already plotted. The center of each circle would lie at the star position and its radius in arc measure would be the same as the measured (and calculated) angular distance from that particular star to the center of the Earth. A unique graphical solution for Earth coordinates, and hence for satellite geographic position, would be given by three such circles. If gross errors occurred in the measurements, they would be quickly detected by this method, and with a minimum of labor. This method is always of value, if only to prevent gross errors.

A variation of this purely graphical solution is to draw these circles on an equidistant star projection. The convenience of a plane chart, as compared to a large globe, is quite obvious.

The second basic method is to find the Earth coordinates in terms of the star distances and known coordinates of the stars by analytical means. One possible solution to this problem by some straightforward applications of solid analytic geometry is outlined briefly below for general reference. The reader might try to supply the missing steps in the development if interested.

The problem may be expressed as follows: given coordinates of the centers, as well as the radii, of three circles on the surface of a sphere, determine if they have a common point, and if so, find its coordinates. It may be shown that this point is the same as the point common to the three planes determined by the circles. Furthermore, where λ, μ, ν are the direction cosines of the radial line passing through the center of one such circle, or of the star, and a is the radius of the circle in arc measure, then the equation of the plane determined by this circle, in rectangular coordinates with origin at the center of the sphere, is given by:

$$\lambda x + \mu y + \nu z - \cos a = 0.$$

The direction cosines of the star can be found from its declination and SHA by the equations given in section 405, if b is taken to be the declination and ℓ the SHA. The intersection of three such planes is then merely all sets of points (x, y, z) which simultaneously satisfy three linear equations similar to that above. If a unique solution exists, the x, y, z coordinates of the unknown point of intersection can be easily converted back to declination and SHA if necessary. This will be left as an exercise for the reader.

Did you supply all the missing steps?

This concludes a brief description of methods which could be used for geographic position determination with a marine sextant, or similar device. It should be pointed out explicitly, however, that the methods will be difficult to apply in navigating very near the Earth. Not only will the Earth's disk blot out a large part of the star field, but, as we have seen in chapter 3, the velocities are very high for close-in satellites. Hence, the angular distances from stars to the Earth's rim will change rapidly with time, making the simultaneous observations of three such angles very difficult. Thus, it becomes necessary for the astronaut to develop techniques for advancing or retarding his measurements to a single time of observation, in the same manner that a marine navigator must allow for the motion of his ship over a period of minutes. Some methods and devices which essentially eliminate this problem, yet have less accuracy than the sextant, are described in the following section.

707. OTHER METHODS FOR POSITION FINDING

Briefly described in the following paragraphs are a few other methods for effecting satellite position measurements by the same basic approach of observing the apparent position of the Earth in the star field. A long discussion of equipment has been avoided in this Handbook because designs change and improve so rapidly.

Photography offers the space navigator some interesting possibilities. It has already been discussed in connection with satellite tracking from Earth stations in chapter 6, and the high accuracy of measurements taken from photographs was pointed out there. If the astronaut were provided with an adequate optical system, he could photograph the Earth's disk against the stars, using, for example, some high-speed Polaroid film, and have the developed picture in about twenty seconds. Even faster and better film should be produced in the next few years. The astronaut could then scale off the declination

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and SHA of the center of the Earth by superimposing a transparent star chart over the photograph, although a number of projections would be necessary. By effectively stopping the apparent motion of Earth within the period of film exposure, this method solves the problem of making simultaneous angle measurements.

An even faster determination of geographic position is possible with an optical system designed by G. T. McNeil of Photogrammetry, Inc. The heart of this device, which is illustrated in figure 707A, is a remarkable concentric lens, sometimes called Foster's eye, developed by the Optical Research Laboratory of Harvard University in 1941. No practical use was found for the lens at that time, because of certain disadvantages, but fortunately they do not apply to this particular design. In using this "space navigation viewer" the astronaut first aligns the optical axis of the system with his vertical so that the image of the Earth's disk is centered in the concentric range circles drawn on the fine grain screen. Then he places a transparent hemispherical star globe over the screen and rotates it until the stars printed on this globe match the images of the actual stars. Finally the position of the optical axis relative to meridians and parallels drawn on the globe gives the geographic position of the satellite. If the markings of these circles are already reversed 180° relative to the stars, position in terms of declination and SHA of the satellite can be read off directly. Four or six of the hemispherical globes, centered at different parts of the star field would be sufficient.

Another space navigation instrument using a star matching technique and a similar concentric lens has been developed by Edwin G. Collen

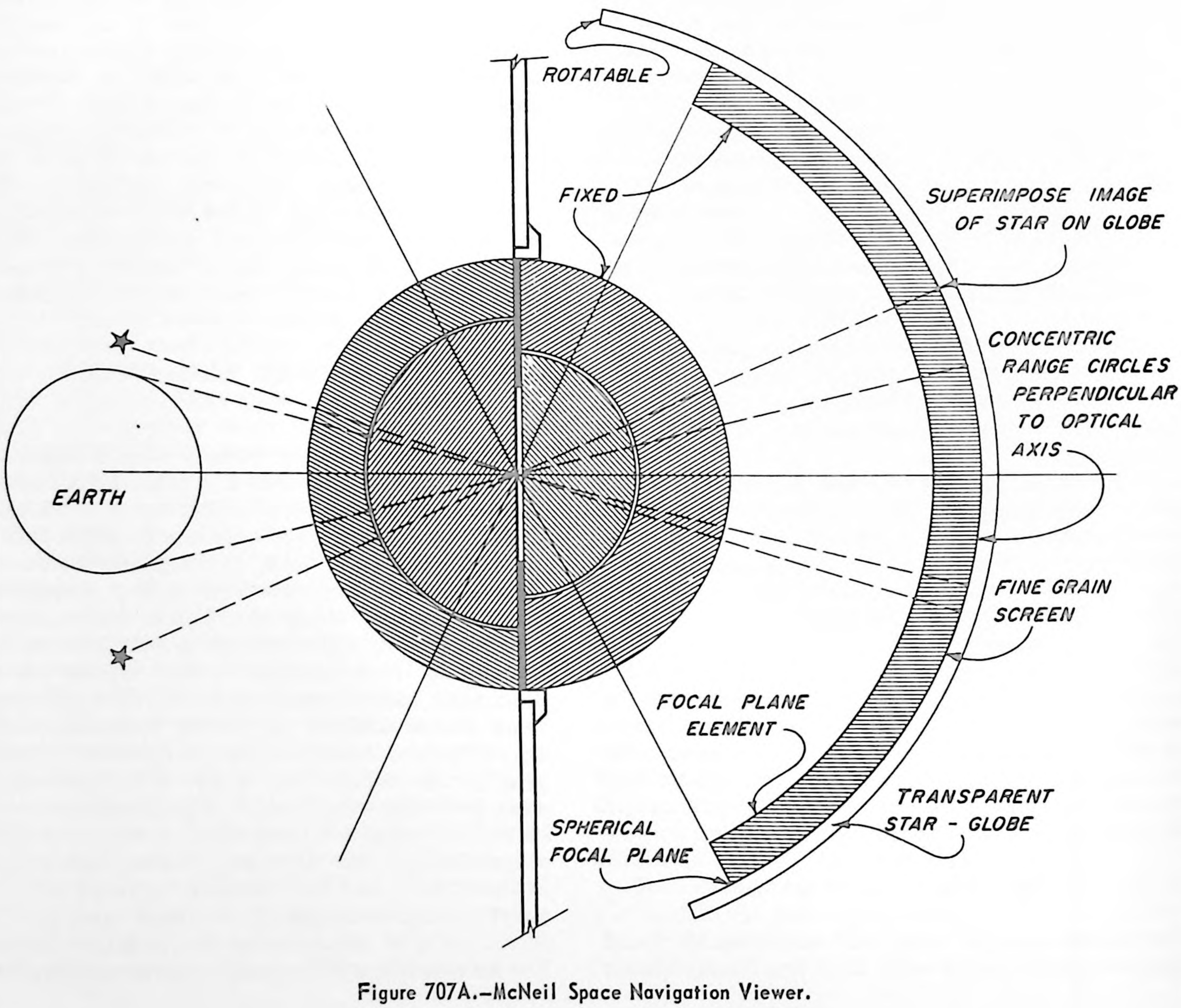

Figure 707A.–McNeil Space Navigation Viewer.

of Kearfott Division, General Precision, Inc. It is a small, hand-held device which permits an astronaut to locate his geographic position on a model Earth globe. A special objective optical system producing an image of the Earth's rim with the background stars, and a reference Earth globe rotating sidereally within a transparent star globe are the two basic components. Figure 707B shows a cutaway drawing of the instrument. Real and artificial star patterns are first matched by manually rotating the star globe while looking through the upper eyepiece. Figure 707C is an artist's conception of what the astronaut would see. The markings on the edge of the field give the altitude of the space vehicle from the observed curvature of the Earth's rim, as suggested in section 705. Geographic position is then read out by viewing the inner globe through the lower eyepiece.

The principle behind this design is illustrated in figure 707D. An observer *C*, at altitude *h*, above point *D* on the Earth sights along the horizon *B*, at a star field *S*. The angle λ defines his altitude. *OC* is the local vertical and *BC* is the tangent to the Earth's rim at *B*. All dimensions are in the plane of the paper. The stars *S*, being at an "infinite" distance, however, are the zenith stars of an observer at *A* on or above the surface. *B* is 90° from *A*, in the direction *A*-to-*C*. The model of the Earth must then be viewed from a point corresponding

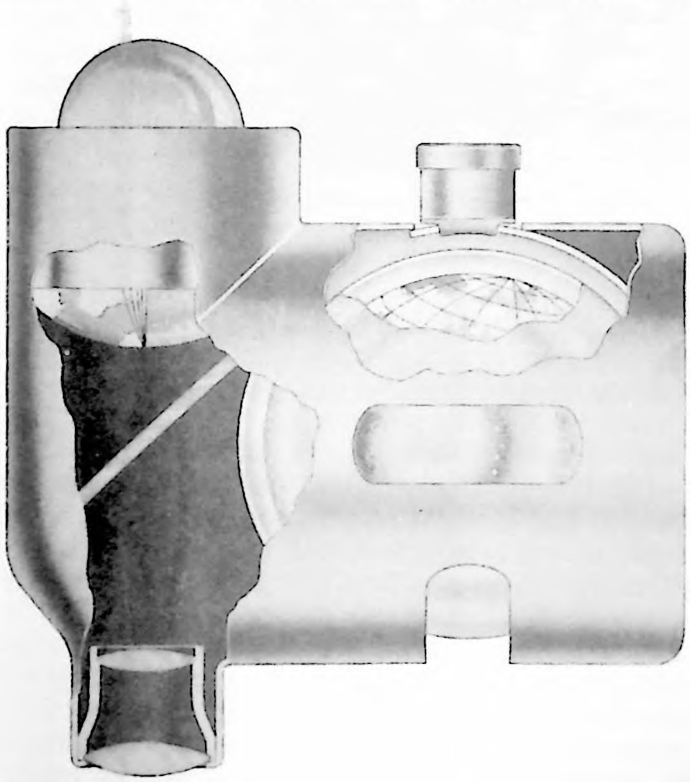

Figure 707B.–Cutaway drawing of Collen Position Finder.

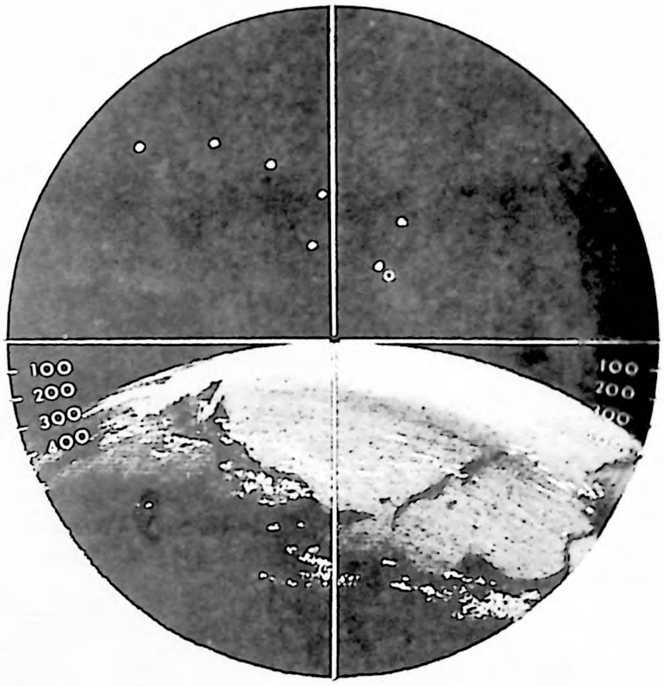

Figure 707C.–Astronaut's View through Instrument.

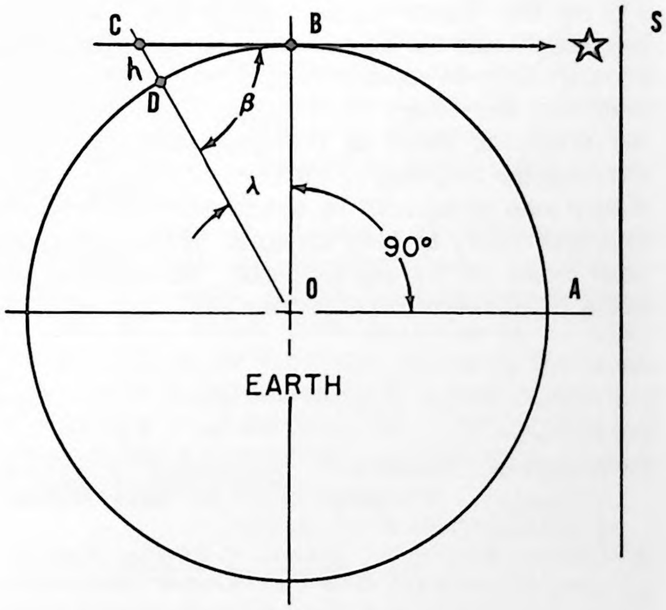

Figure 707D.–Principle of the Position Finder.

to *C*, and the coordinates of *D* will then be apparent.

The components of this instrument are arranged as in figure 707E. The astronaut sights through the eyepiece on a star field near the horizon along the line CBS. The Foster objective lens forms a real, inverted, and reversed image of the stars and a portion of the horizon

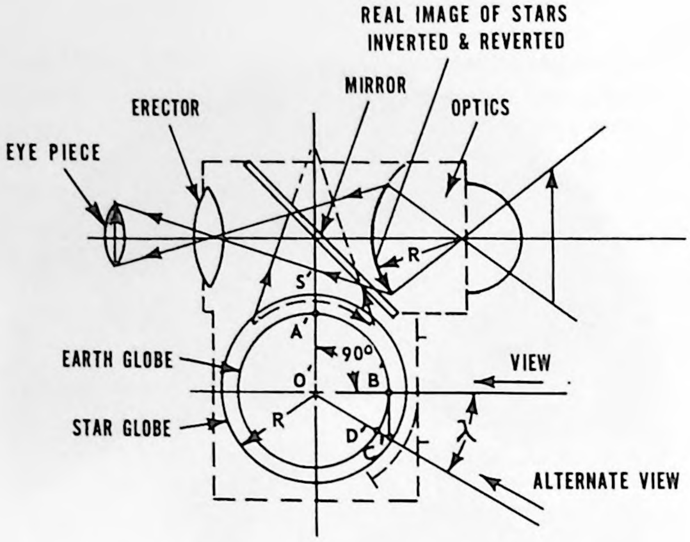

Figure 707E.–Arrangement of Components in the Instrument.

on the spherical screen of radius R. This real image is seen through the eyepiece, an erecting lens, and a beam-splitter mirror. This mirror also reflects an image of the exterior of the star globe, also of radius R, surrounding the Earth globe. The artificial stars, made of a fluorescent substance and illuminated by ultraviolet light, are those zenith stars, S', above a point A', on the Earth globe. After the observer manually turns the Earth-star globe combination around its own center within the instrument, matching the image of the real stars to that of the artificial stars on the globe, point A' will correspond to point A on the real Earth. Any distortions which may be created by the erector eyepiece apply to both images. If the observer then looks at the Earth globe, illuminated by white light, from the direction $B'O'$ through the reticle crosshairs, he reads the coordinates of his geographic position for sea level altitude. But for altitudes above sea level this reticle must be moved circumferentially about O' through the angle λ. If this Earth-viewing eyepiece moves along a scale calibrated for altitude in miles, the angle λ need not be determined explicitly. Indeed the satellite altitude has already been found from the scale shown in figure 707C.

708. NAVIGATION BY TERRESTRIAL OBSERVATIONS ONLY

We have now described a number of methods by which the astronaut might determine his geographic position, all based upon the apparent position of the Earth in the star field. Before concluding this brief discussion of satellite-based tracking techniques, it should be pointed out that latitude and longitude can be determined under certain circumstances without reference to the stars at all. When land areas on the Earth's surface are visible and clearly defined, as they should be for most cislunar navigation, the coordinates of the satellite's geographic position can be determined directly by locating the center of the Earth's disk relative to the land areas, and comparing with a chart. The quickest method would be to superimpose a small but very accurate globe on the observed Earth, and read off the latitude and longitude from the globe. An optical design similar to those of McNeil and Collen, described in the preceding section, could be used to effect this direct measurement. The accuracy of such a method would depend upon the design itself, and mainly on the size and detail of the globe used.

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CHAPTER 8

PREDICTING SATELLITE POSITIONS

Rocket technology has now advanced to the stage where it is possible to place a manned space vehicle into orbital flight. In the near future, space travel will be largely restricted to motion of free fall, after a short period of powered flight from the launching station on Earth. As space navigation techniques are perfected, astronauts will have the ability to change their orbits within certain restricted limits. When propulsion is improved, these limitations will be slowly overcome, and finally the space craft will be able to escape from the Earth's gravitational field and follow whatever course may be desired.

801. SPACE DEAD RECKONING

The term "dead reckoning," a familiar corruption of "deduced reckoning," might well be used to denote the technique of predicting the positions of a satellite vehicle in space. As long as we restrict ourselves to orbits of powerless flight, the methods of determining positions of satellites from observational data are well known among astronomers. In chapter 4 the techniques for determining orbital elements were described. The construction of an ephemeris, or a set of predicted positions for given times, forms the second major part of the tracking problem. The same methods can be applied to satellite vehicles orbiting Earth as are used to keep track of the thousands of minor planets orbiting the Sun.

The techniques of so-called space dead reckoning, which are described in this chapter, fall into two categories—one in which the orbital elements of the satellite are given, and another in which a series of fixes have been determined, say, by the methods of chapter 7. This latter technique is emphasized because of the extent to which it simplifies the work of the space navigator. Indeed, when a few more positions are known, the intermediate task of determining orbital elements by the analytical methods of chapter 4 can be shortened considerably.

802. POSITION FROM THE ORBITAL ELEMENTS

The construction of an ephemeris, given the orbital elements of a satellite body, can be a very laborious task because of the fact that the technique employed must be repeated many times. One selects a particular value of time, considered as the independent variable, and by one of several available methods[1] finally determines the position of the satellite at this time in whatever coordinates may be desired. Then another value of time is chosen, and the same process is repeated. For the relatively slow moving planets of the Sun, the interval between the calculated positions can be considerable. Interpolation techniques then fill in the gap. However, with fast-moving Earth satellites this is not always possible or desirable. Because of the repetitive nature of ephemeris construction, the problem naturally lends itself to machine computation.

One particular method[2] of determining an ephemeris will now be described, assuming the orbital elements are in the form presented in section 402. Either from the value of the semimajor axis a, or from the values of the semiparameter p and eccentricity e, one determines the period P of the satellite. The relationship between these quantities was derived in chapter 3, and is given by the equation

$$P = 2\pi a \sqrt{\frac{a}{K}}$$

which may be solved either by a set of tables, or by an alignment chart or nomogram[6] such as the one shown in figure 802A. Then the mean daily motion

$$\mu = \frac{2\pi}{T}$$

is calculated. The value of time, the independent variable, is substituted into the equation $M = \mu(t - t_p)$ to determine the instantaneous

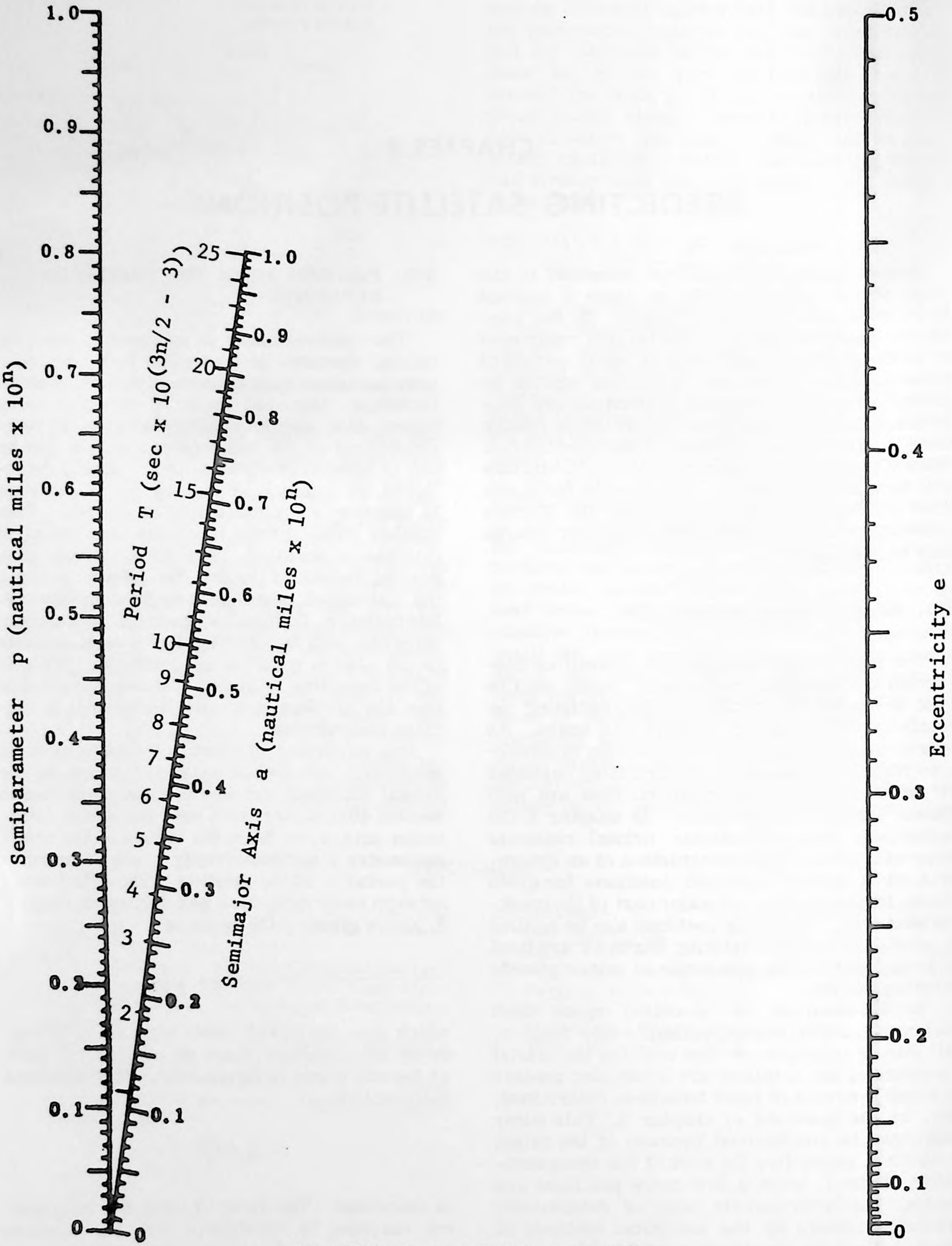

Figure 802A.—Nomogram Relating p, a, e and T.

mean anomaly M. This is simply the angular coordinate measured from the perigee point to the position of a fictitious body which orbits the central body at a uniform angular rate, and having the same period as the real body. Knowing the value of M, one must determine next the eccentric anomaly E by solving Kepler's equation

$$M = E - e \sin E.$$

This equation is of course highly transcendental, and can be solved for E only approximately, either by tables, by iterative numerical methods to any desired accuracy, or by a nomogram[6] such as the one shown in figure 802B if extreme

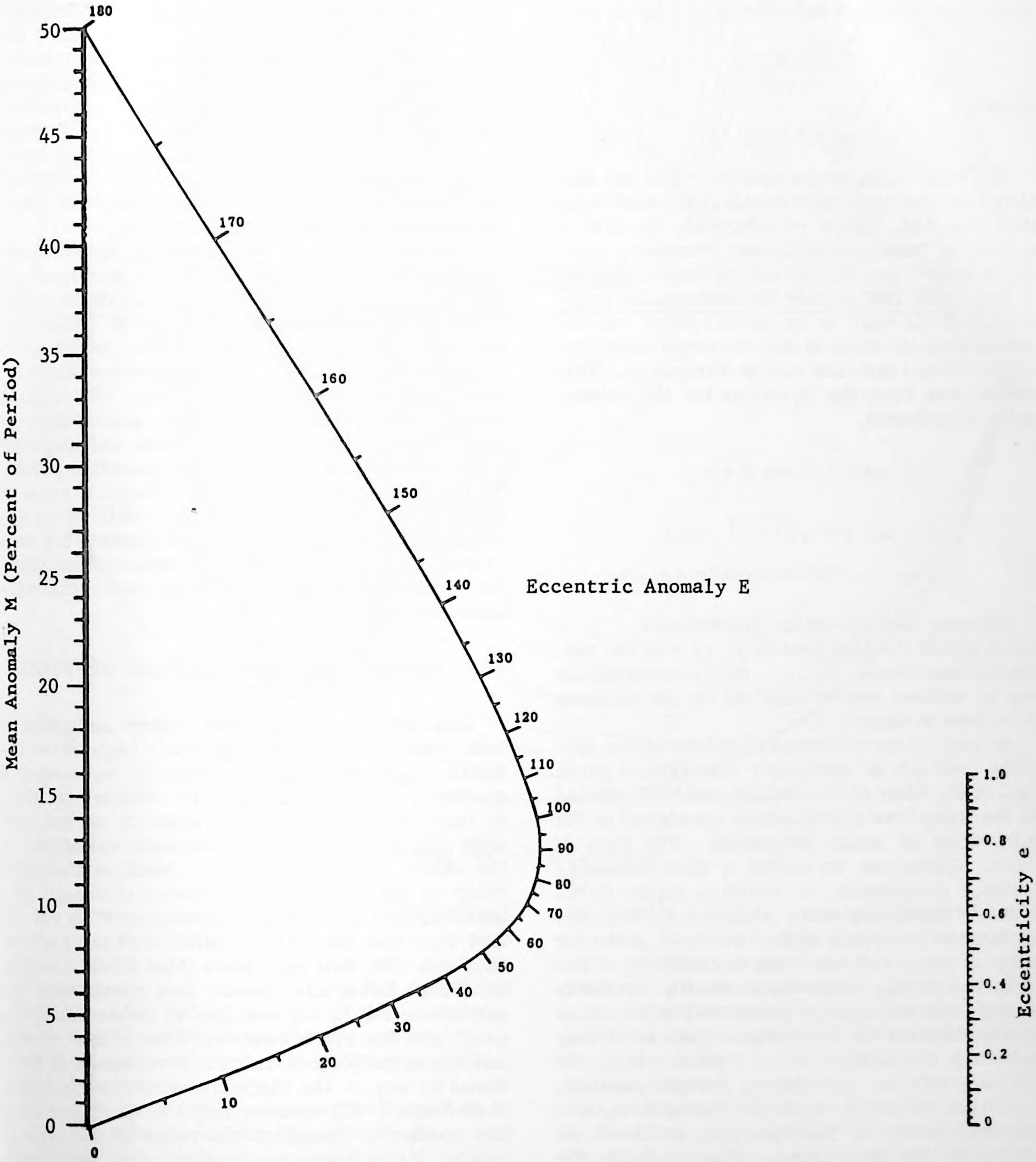

Figure 802B.–Nomogram for solution of Kepler Equation.

accuracy is not so important. Finally, the true anomaly θ can be determined as a function of E by one of several relationships, such as

$$\tan \frac{1}{2} \theta = \sqrt{\frac{1+e}{1-e}} \tan \frac{1}{2} E .$$

The radial distance r, if desired, can now be determined in one of several ways. For example

$$r = \frac{a(\cos E - e)}{\cos \theta} ,$$

or better

$$r = a(1 - e \cos E) .$$

The focal polar coordinates (r, θ) of the satellite have now been determined at the particular time selected. There remains only the task of converting these into whatever reference coordinate system may be desired. It should be noted at this point that, if only the rectangular coordinates of the body in its orbital plane are required, then the steps of determining true anomaly and radial distance can be eliminated. This can be seen from the equations for the rectangular coordinates,

$$x = r \cos \theta = a \cos E - e ,$$

$$y = r \sin \theta = a\sqrt{1 - e^2} \sin E ,$$

$$z = 0 .$$

Knowing the value of the three angles i, Ω, ω, which orient the coordinates x, y, z to the reference coordinates X, Y, Z, the transformation one to another can be effected by the methods described in chapter 5.

So long as the predicted positions of the satellite need not be extremely accurate, a great deal of the labor of this method can be shortened by the use of two coordinators developed in the pilot class of space navigation. The first of these, which may be called a time/anomaly/distance coordinator, is shown in figure 802C. It solves the multiple step problem of determining true anomaly and radial distance, given the value of time and the orbit eccentricity. The form shown was based upon readily available tables[3] and the concept presented in a lecture to the class by G. D. Dunlap. One need only calculate the percent of a period which has elapsed since the most recent perigee passage, select the value of e on the radial arm, and rotate this arm so that this point on the scale intersects the time curve. The value of true anomaly can then be read directly from the semicircular scale, and the radial distance factor determined from the set of curves in the lower half circle. This factor, when multiplied by the range at perigee, gives the radial distance at the time selected.

The second of these devices is shown in figure 802D, and may be called a spherical coordinator. This graphical computer was designed by G. D. Dunlap, Annapolis, Maryland. The plastic overlay can be rotated about the origin (which represents the ascending node), and is marked with a single line representing the plane of the orbit. The linear scale along this line gives the argument of the latitude $u = \theta + \omega$. Underneath is an azimuthal equidistant projection of a sphere, with the latitude scale running vertically, and the longitude along the horizontal equator line.

The procedure is quite simple to determine the geocentric coordinates of the satellite once the true anomaly is found by the methods above. The transparent plastic overlay is rotated so that the index at one end of the linear scale points to the value of the inclination angle i, and the value of u is then plotted. The location of this point is estimated on the azimuthal projection below in terms of latitude and longitude past the ascending node (or meridian angle). To this latter angle must be added the value of longitude of the ascending node itself. This essentially solves the problem of locating the satellite at the selected time. A three-dimensional fix can be given by calculating the radial distance as well.

803. POSITION FROM A SERIES OF FIXES

One fact in favor of the space navigator is that, once in an orbit sufficiently high above the Earth that atmospheric drag is no longer a serious problem, his satellite vehicle continues in that orbit at a nearly constant period, and with only small, fairly predictable variations in the other orbital elements. Most of the difficulty of determining the orbital elements of a satellite by the methods of chapter 4 lies in the fact that the absolute minimum of information is given. We will now show that when a series of three fixes are given, one more than the minimum required, the job of determining the orbit can be much easier. The points of this orbit may be plotted from a succession of fixes found by any of the methods described in chapters 6 and 7. Of course, only two such positions are needed to establish the plane of the orbit in space. If the angular coordinates of these three positions are plotted on a sphere and a great

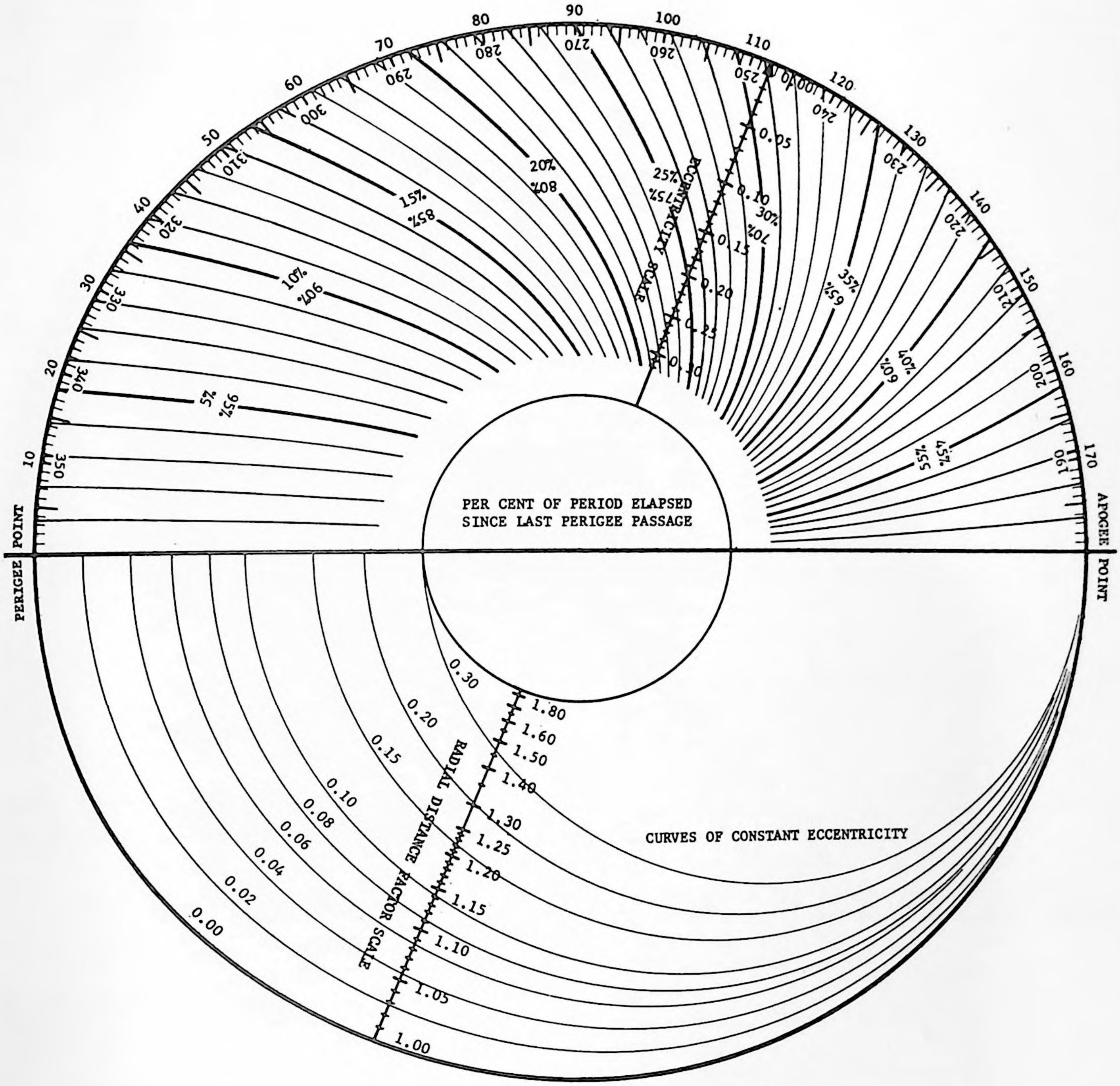

Figure 802C.—TAD Coordinator.

circle drawn through them, the angles which orient the orbital plane to the reference system can be immediately measured with only a spherical protractor and a scale along the equator of the globe. Also the arguments of the latitude u, for each of these three positions, can be immediately measured from the globe plot with a great circle scale.

We now have the necessary information to solve the problem within the orbital plane, and determine essentially three more of the orbital elements by methods much shorter and more straightforward than those of chapter 4. The problem is merely that of determining which conic section passes through the three given points in the plane when the points are given with respect to the focus. Knowing the three radial distances and the three angular coordinates (arguments of the latitude), the semimajor axis and eccentricity of the conic, as well as the angle which specifies its orientation in the plane with respect to some reference direction (here the ascending note) can be determined.

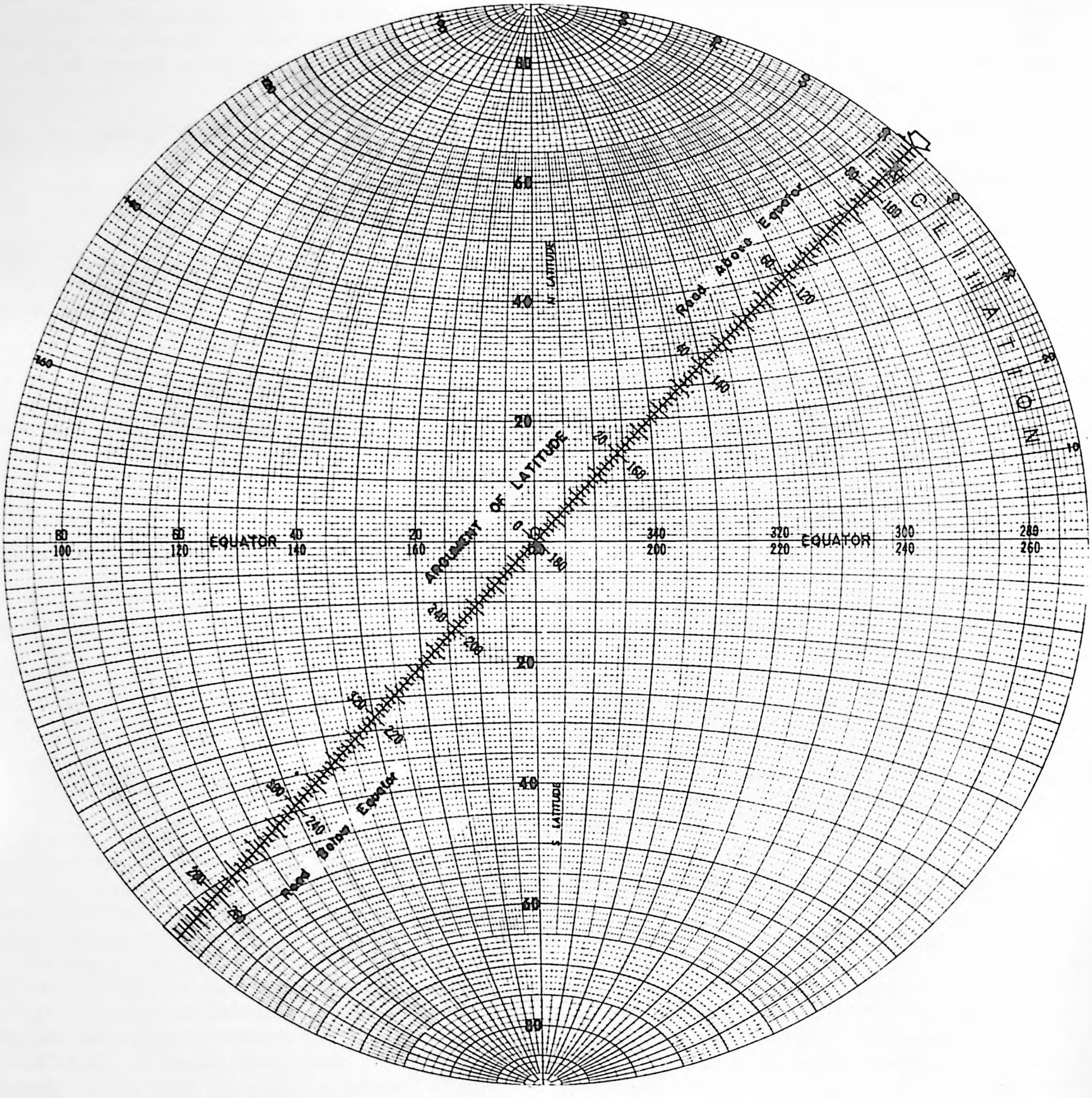

Figure 802D.–Spherical Coordinator.

We begin with the general equation of the conic

$$r = \frac{p}{1 + e \cos \theta} = \frac{p}{1 + e \cos (u - \omega)}$$

where $p = a(1 - e^2)$.

Figure 803 illustrates the three given points $p_1(r_1, u_1)$, $p_2(r_2, u_2)$, $p_3(r_3, u_3)$ and shows the conic (ellipse in this case) which passes through them and must be determined. To find the three parameters p, e, ω, we begin by writing

$$r_i = \frac{p}{1 + e \cos (u_i - \omega)}, \qquad i = 1, 2, 3.$$

Next we eliminate p with the result that

$$r_1\left[1 + e\cos(u_1-\omega)\right] = r_2\left[1 + e\cos(u_2-\omega)\right] = r_3\left[1 + e\cos(u_3-\omega)\right]$$

$$r_1 - r_2 = e\left[r_2\cos(u_2-\omega) - r_1\cos(u_1-\omega)\right]$$

$$r_1 - r_3 = e\left[r_3\cos(u_3-\omega) - r_1\cos(u_1-\omega)\right].$$

Then e can be eliminated by the division of these last two equations, resulting in the ratio

$$a = \frac{r_1 - r_2}{r_1 - r_3} = \frac{r_2\cos(u_2-\omega) - r_1\cos(u_1-\omega)}{r_3\cos(u_3-\omega) - r_1\cos(u_1-\omega)}.$$

The cosines of the difference of the two angles can be expanded giving

$$a = \frac{\cos\omega\,(r_2\cos u_2 - r_1\cos u_1) + \sin\omega\,(r_2\sin u_2 - r_1\sin u_1)}{\cos\omega\,(r_3\cos u_3 - r_1\cos u_1) + \sin\omega\,(r_3\sin u_3 - r_1\sin u_1)}$$

which will be abbreviated as

$$a = \frac{x_1\cos\omega + x_2\sin\omega}{x_3\cos\omega + x_4\sin\omega}$$

where x_1, x_2, x_3, x_4, and a are all known. Cross multiplication yields

$$(ax_3 - x_1)\cos\omega = (x_2 - ax_4)\sin\omega$$

and

$$\omega = \text{Arc tan}\left(\frac{ax_3 - x_1}{x_2 - ax_4}\right).$$

After ω has been determined by this equation we can next find the eccentricity from the equation

$$r_1 - r_2 = e\left[r_2\cos(u_2-\omega) - r_1\cos(u_1-\omega)\right] = e\left[x_1\cos\omega + x_2\sin\omega\right].$$

Finally the semiparameter p can be determined from

$$r_1 = \frac{p}{1 + e\cos(u_1-\omega)}.$$

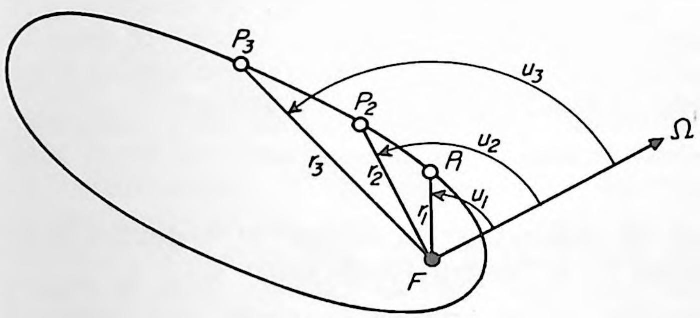

Figure 803.–Drawing of three three-dimensional fixes in orbit plane.

Once these parameters of the orbit have been determined, of course, the period is known by the method of the preceding section. Knowing the value of the argument of the latitude at the last known position, one determines the true anomaly at the last known position, and from the time/anomaly/distance coordinator can determine the percent of the period which has elapsed since perigee. Then by noting the time since the last known position, the new true anomaly for any instant can be calculated with the same device. In this way the satellite vehicle's position in the orbit can be predicted for any time, or the time can be determined when the satellite will be at any point in the orbit. This method is analogous to dead reckoning a ship at sea. It has the advantage of being completely flexible, allowing for the varying angular velocities in an elliptical orbit, as well as parabolic and hyperbolic orbits, since the value of e in the equations above might take on any value. The method obviously is much shorter than that of Gauss.

Considering the fact that a space vehicle will have the same orbital elements over a fairly long period of time, the short time required to determine three fixes, to plot them on a globe, and to calculate the necessary information does not appear to be too lengthy a task. In fact it is likely that the astronaut in some cases will have a good deal of time to devote to such navigation calculations. It is entirely possible that for nearly circular orbits, he will be inclined to plot his entire orbit rather than to compute and predict positions by these methods.

804. PLOTTING ON A GLOBE

It has been pointed out above how a satellite vehicle's position may be plotted on a globe such as that of McMillen, and the angles which orient the plane of the orbit to the reference coordinate system determined directly. The arguments of

the latitude for each of the positions likewise may be measured directly without calculation. With the McMillen globe, careful plotting can result in an accuracy of at least 10 minutes of arc for angular measurements along a great circle, and $1/2°$ for the angles which must be measured with a spherical protractor.

Another type of globe might also be used. Suppose the observer is situated with his eye at the center of a transparent star globe on which are printed meridians and parallels, so that when the globe is properly oriented each star on it coincides with the corresponding real star in the heavens. Then the coordinate grid would be seen in its proper position in the star field. With this sort of globe, correctly oriented, any position in the heavens within a certain range of arc may be read directly from the globe. A succession of positions for the Earth's center marked on the star globe will trace the path of the satellite vehicle through the heavens. When the times for these positions are also marked on the globe, the observer should be able to predict an approximate time of arrival for any point in the orbit, provided it is a fairly circular one. The limited accuracy of such a plot, however, is a big disadvantage.

The same type of device might be used for elliptical orbits as well, except for the handling of variable angular velocity. This would again require the time/anomaly coordinator.

805. PLOTTING WITHIN THE ORBITAL PLANE

Since the plot on the surface of a sphere cannot be conveniently used to show radial distances in addition to the two angular coordinates, a plot of the orbit on a plane surface is also quite useful. As we shall see in the following section, the plane of the orbit of a satellite of the Earth precesses with respect to sidereal coordinates. In other words the sidereal hour angle of the ascending node changes with each revolution of the satellite. So when the actual orbit is plotted on a globe, it does not come around and form a closed figure. Still the arguments of the latitude for each position can be determined with fair accuracy from the globe plot, and these arguments together with radial distances for each of the fixes transferred to a plot of the plane of the orbit. Then within this plane the satellite path remains more nearly constant. However, again for most orbits, the position of the perigee changes, or the ellipse precesses around in its plane. But this effect is usually much smaller than the motion of the node. Consequently, a plot of the orbit within its plane does essentially form a closed figure and need be plotted less often.

806. MAJOR PERTURBATIONS OF THE ORBIT

All of the calculations of orbits made thus far have been based upon the assumptions of chapter 3, that the body was moving in a perfect central force field and in a perfect vacuum such that no other forces exist. In this idealized case, as we have seen, the orbit has the form of a perfect conic section with a fixed orientation in space and translating with the center of mass.

Unfortunately, this idealized case does not exist,[4] so such a conic section is not the best description of the real satellite's orbit around the Earth. The Earth and satellite in question are not alone in space, so the gravitational attractions of other bodies, particularly the Moon, must be considered. The Earth has an atmosphere which at low altitudes seriously affects the satellite motion, dissipating its kinetic energy through drag and eventually causing complete decay. The satellite body has a finite cross section to the pressure of solar radiation, and so it experiences another force on this account. Finally, a perturbation which usually has the largest effect for satellites above 100 miles, and with which we are most concerned in this section, stems from the fact that the gravitational field of the Earth does not have spherical symmetry. Since the Earth itself is not a perfect sphere, the gravitational force on a body orbiting around it varies in both magnitude and direction for different points at the same distance away from the center. Figure 806A exaggerates the equatorial bulge to explain its effect. The Earth can be approximated mathematically by an oblate spheroid (onion shape), but even this has been shown to be a poor approximation from recent analyses of transit satellite orbits. According to these studies, even the equator is not circular.

If we use this oblate spheroid description of the Earth to deduce a more correct satellite orbit, we essentially replace the central force expression for the potential energy,

$$U = GM/r,$$

by an expansion[5] in terms of Legendre functions P_n of the colatitude angle ϕ,

$$U = \frac{GM}{R}\left[\frac{R}{r} - \sum_{n=2}^{\infty} J_n \left(\frac{R}{r}\right)^{n-1} P_n(\cos\phi)\right].$$

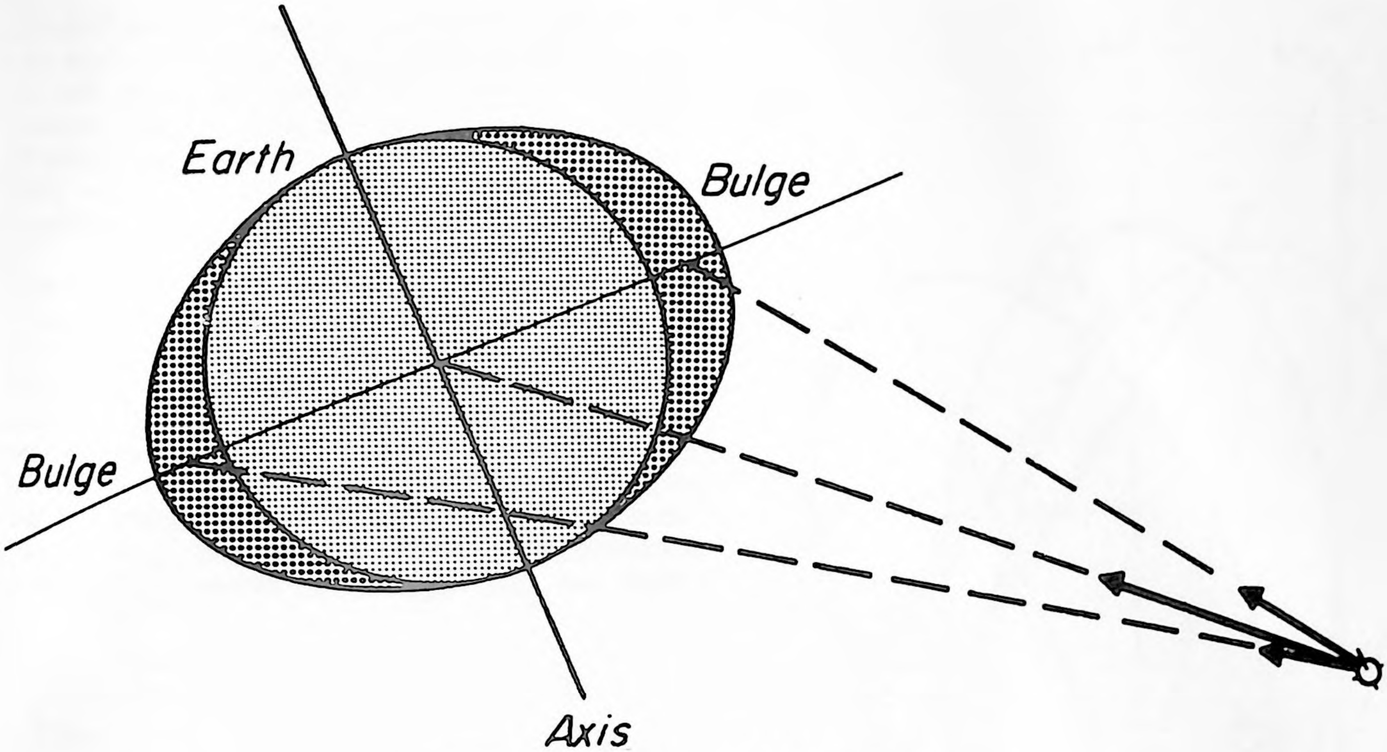

Figure 806A.–Effect of Earth's Equatorial Bulge.

The constant coefficients J_n can be determined from observational data, and according to recent analyses have the approximate values

$$J_2 = 0.0016238,$$
$$J_3 = 0.0000022,$$
$$J_4 = 0.0000014.$$

If the equations of motion of the satellite are solved using this expression for the potential energy, two essential differences result in the form of the orbit. Assuming bounded motion, such that the satellite does not escape the Earth, the path within the orbital plane can be approximated by a precessing ellipse. That is, the position of the perigee point of its "osculating" ellipse rotates around with time. This advance of the perigee is illustrated by figure 806B. The actual path of the satellite is indicated by the solid curve. The osculating elliptic reference orbit is shown for the positions 1 and 3 of the body. It is the orbit the satellite would follow if the perturbations stopped when the satellite reached that position. The other principal change due to the Earth's bulge is in the orientation of the satellite plane with respect to sidereal coordinates. If the satellite is moving predominantly to the east, that is, with an inclination angle less than 90°, it will cross the equator at each passage farther west than the preceding one. This regression of the line of nodes is in addition to the apparent westward motion due to the eastward rotation of the Earth. In other words, the sidereal hour angle of the ascending node increases with each revolution for such orbits. When the inclination exceeds 90° the reverse is true. For polar orbits where $i = 90°$, this motion of the nodes essentially stops.

Figure 806C furnishes a simplified explanation of this phenomenon. Suppose the satellite is travelling along a path from X to Y such that it crosses the equator at Ω_1. Now let the added attraction of the equatorial bulge be simplified into an instantaneous force that acts on the body when it reaches the point M. As a result it crosses the equator at Ω_2 instead, proceeds to point N where a compensating force from the bulge diverts the satellite into the path NZ, which crosses the equator at Ω_3. The effect of the perturbation is actually a gradually changing one, rather than two instantaneous accelerations, so the ascending node accordingly regresses from Ω_1 to Ω_3 gradually during the satellite's period.

To a first approximation both of these changes are fairly linear with time, and expressions for the angular rates are given below.

$$\dot{\omega} = \frac{2\pi JR^2\left(2 - \frac{5}{2}\sin^2 i\right)}{p^2},$$

$$\dot{\Omega} = \frac{+\ 2\pi JR^2 \cos\ i}{p^2}.$$

of an orbit having an inclination less than this critical angle moves in the direction of the satellite's motion, and above this value the reverse is true. Both of these secular precession rates can be determined approximately with the nomograms[6] shown in figures 806D and 806E. Finally the effect of atmospheric drag is illustrated by figure 806F.

If the satellite is in the orbit AP and the drag is concentrated in a small region near the perigee P, the velocity will be decreased in this region. Therefore the vehicle will not quite reach the same apogee, but will instead pass through a lower point A'. Since the semimajor axis is diminished, the period will be also, in accordance with Kepler's third law, and the satellite will return to P sooner than it would otherwise. It may at first seem strange that a

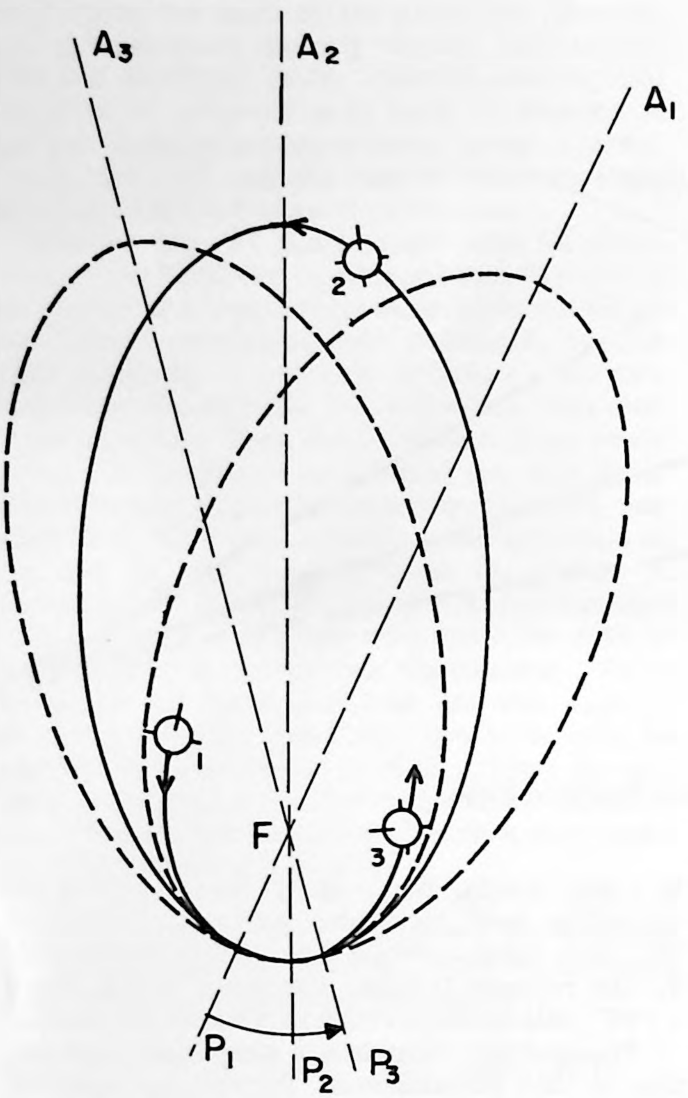

Figure 806B.–Explanation of Advance of Perigee.

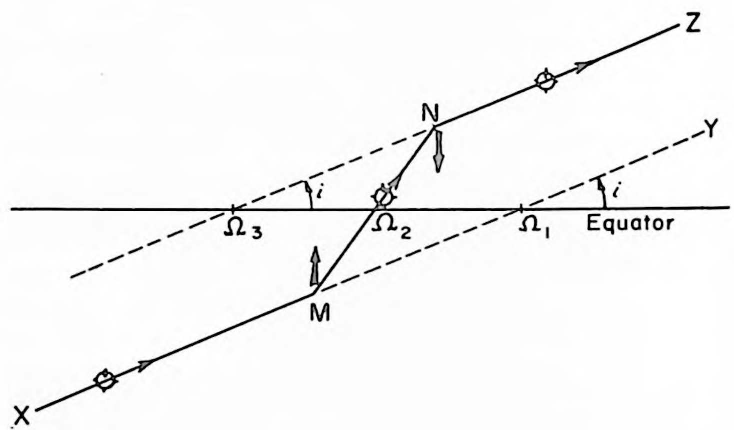

Figure 806C.–Explanation of Precession of Node.

$\dot{\omega}$ is the rate of change of the argument of perigee and $\dot{\Omega}$ the rate of change of the sidereal hour angle of the ascending node, both in radians per revolution of the satellite. To this approximation the rates of change depend only upon the size of the ellipse, and the inclination angle of the orbital plane. Note that if the inclination angle is such that

$$2 - \frac{5}{2}\sin^2 i = 0,$$

or about 63.4°, then the precession of the perigee within the orbital plane stops. The perigee

decrease in the satellite velocity at P results in an increase in the average velocity, but this is indeed the case. The gradual changes in a satellite period can be fairly well predicted from sufficient observational data, and this rate of change is often given with the orbital elements.

The approximate lifetimes of Earth satellites can be determined[4] from the height and velocity at perigee. A graph giving such information is illustrated in figure 806G. It is evident that atmospheric drag becomes a serious problem when the perigee point is below about 100 miles.

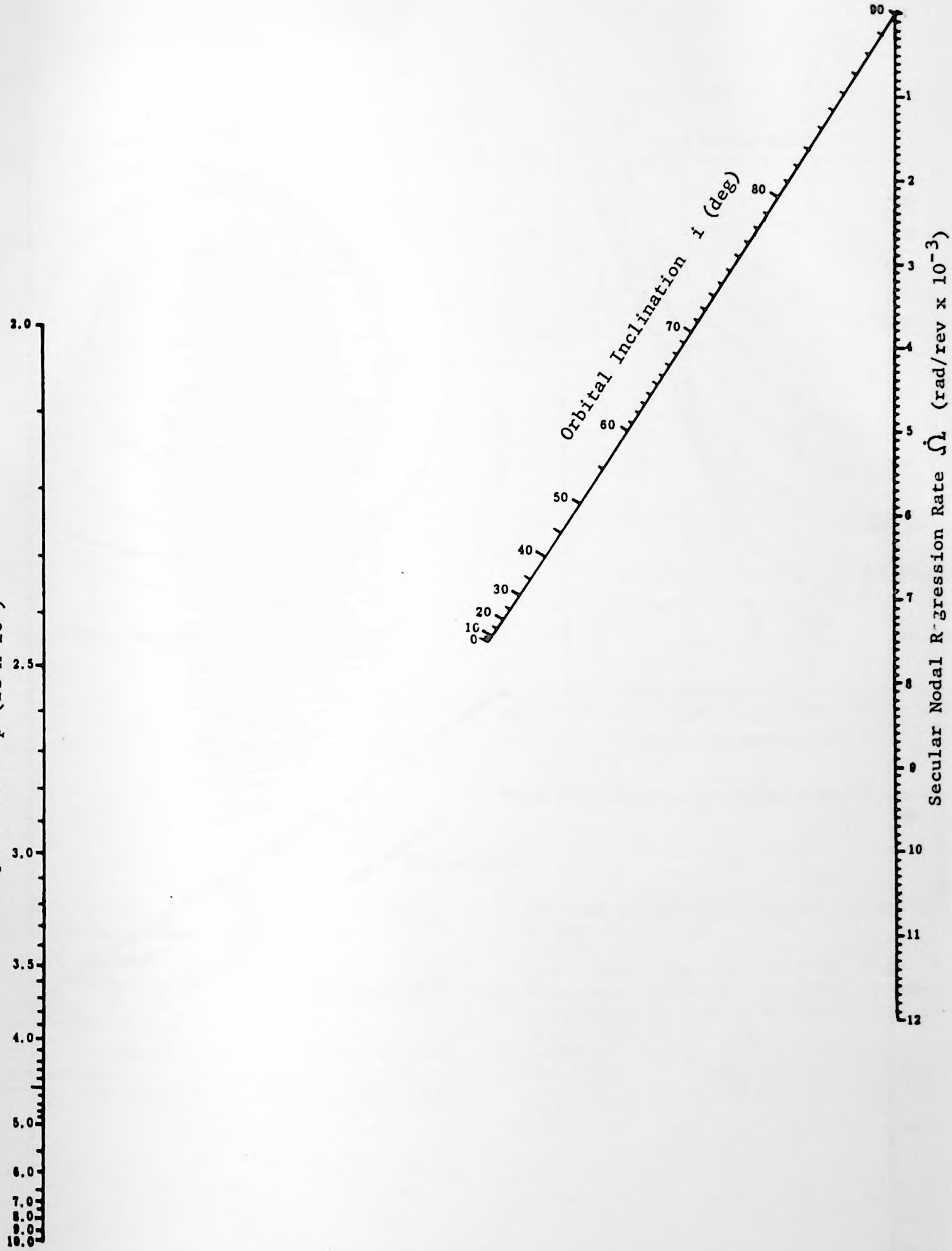

Figure 806D.–Nomogram for solution of Nodal Precession Rate.

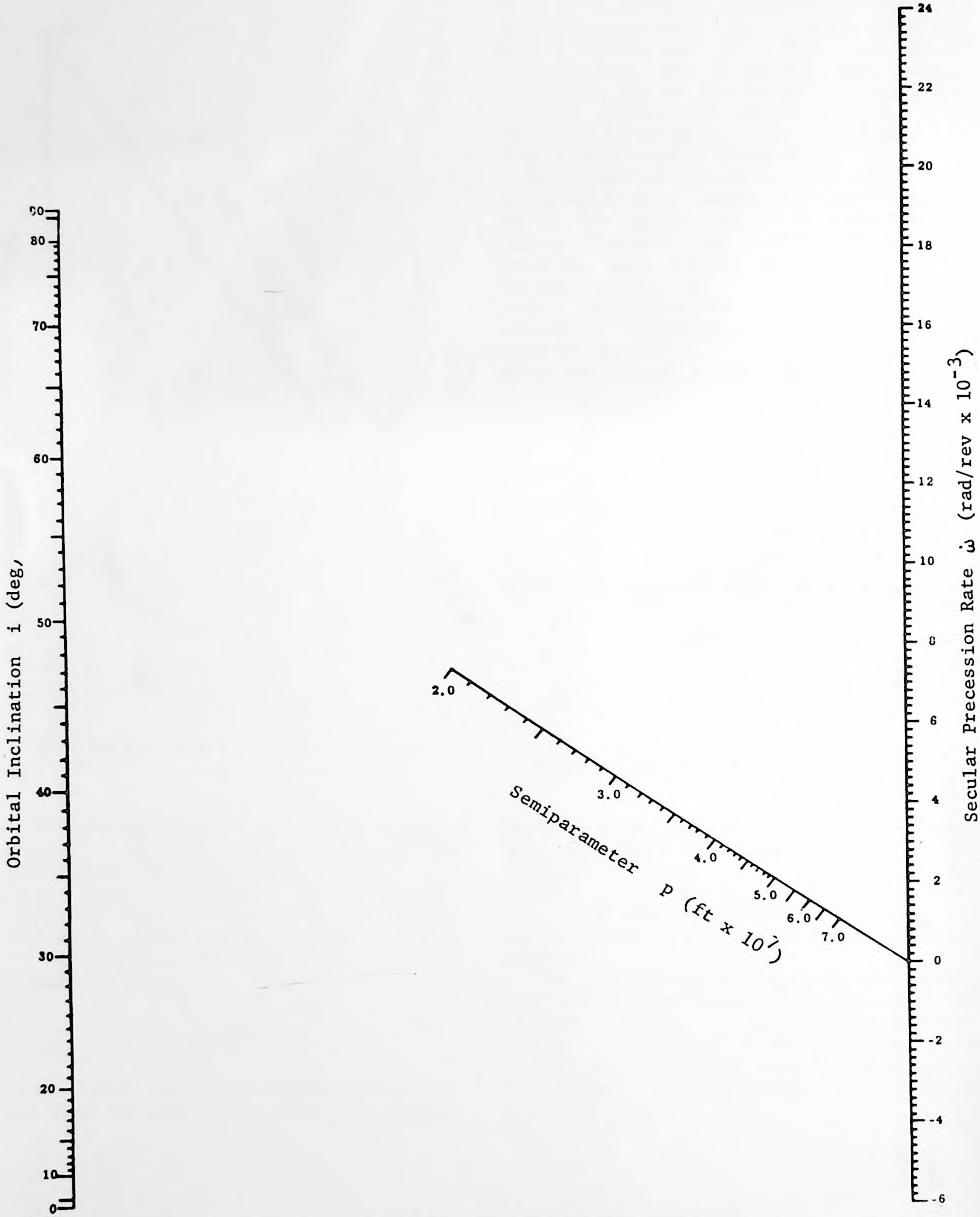

Figure 806E.–Nomogram for solution of Perigee Precession Rate.

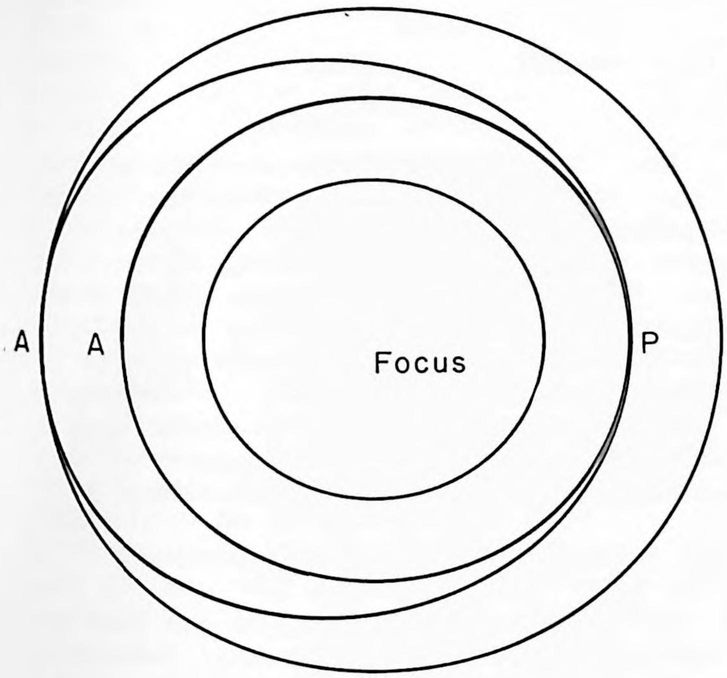

Figure 806F.–Drag Effects on Satellite Orbit.

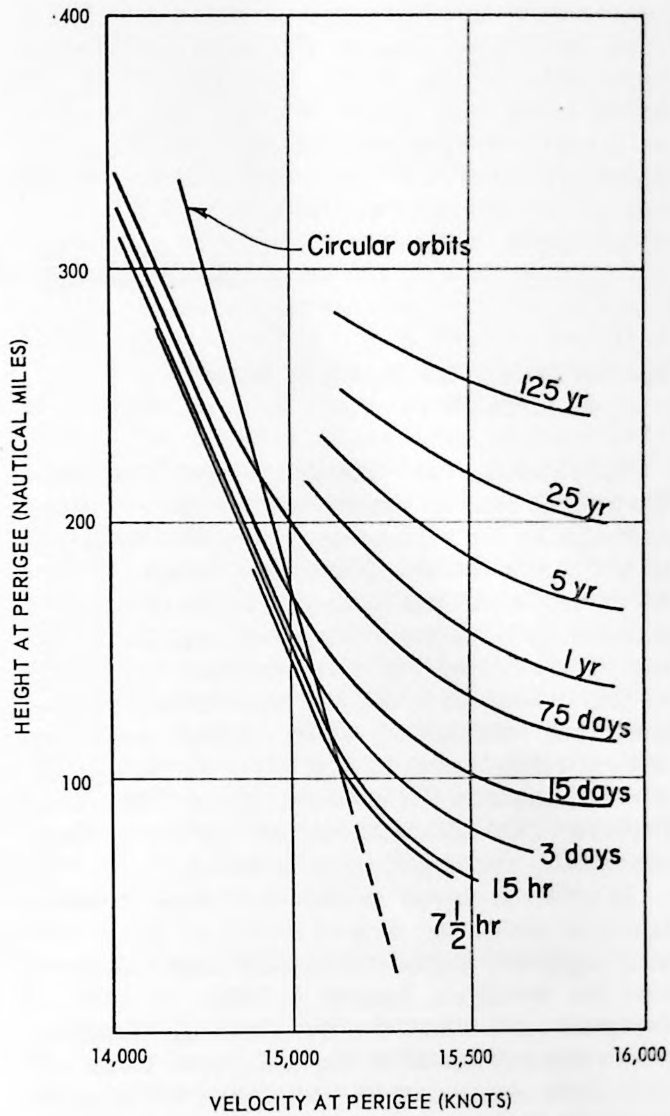

Figure 806G.–Graph for Determination of Satellite Lifetime.

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CHAPTER 9

A SPACE ALMANAC

901. HISTORY OF NAVIGATIONAL ALMANACS

Ephemerides and almanacs have long been the helpful tool of the navigator. Early tables compiled by the astronomers listed the positions of the stars and the planets in terms of right ascension and declination. The first nautical almanac, a publication designed especially for use by the navigating mariner, was introduced by the British in 1767, and was followed by an American version in 1855. Other than the change from astronomical time to Greenwich Mean Time and the deletion of the old Lunar Distance Tables, the almanacs have remained essentially unchanged to the present.

In 1928 the senior author suggested the tabulation of positional data in terms of Greenwich hour angle and helped develop the Lunar Ephemeris for Aviators, issued in 1929. In 1933 he designed the format for the first Air Almanac, which was published by the U. S. Naval Observatory. The Air Almanac was combined with the Nautical Almanac the next year, but a similar publication was issued in Great Britain in 1936. The Air Almanac, as we know, was reintroduced by the Navy in 1941, and has since become a great aid to both air and marine navigation. It is currently published in a joint effort by the American and British governments.

902. NEED FOR SPACE ALMANAC

As we enter the space era, we will be confronted with new and different problems in navigation. Neither the Nautical Almanac nor the Air Almanac will suit the needs of the astronaut. Since he is moving in an orbit with the Earth as the central body, he will want to know his position in space relative to the center of the Earth. We have already discussed techniques by which the space navigator can find the apparent position of the Earth's center projected on the celestial sphere. Therefore, the information of most use to him will be an ephemeris of the Earth as seen from the satellite.

Whereas the Nautical and Air Almanacs give ephemerides of many bodies in the heavens, the Space Almanac should tabulate the position of only one, the Earth. Since the satellite travels in a unique path about the Earth, a path determined by the orbital parameters, we need give data for only a single orbit. Variations of the orbital parameters caused by the departures from sphericity of the Earth are predictable to a great extent, and would, of course, be compensated for in the almanac computation.

903. PROPOSED SPACE ALMANAC

Once in an orbit sufficiently far from the Earth that position finding by the methods of chapter 7 becomes practical, the astronaut will be primarily interested in the Earth's apparent position as seen from the satellite, just as the marine navigator is primarily interested in the apparent position of the Sun as seen from the Earth. Therefore it is proposed that the Space Almanac give, for the predicted satellite orbit, or for selected orbits in a narrow range about the predicted one, the coordinates of the Earth's center in a frame with its origin at the satellite. Celestial equatorial coordinates are the preferred reference system, because they are familiar to most navigators, because the satellite orbit is almost fixed in space with respect to these coordinates, and because the transformation to terrestrial latitude and longitude is easily accomplished.

Thus, it is recommended that the Space Almanac give the declination and sidereal hour angle of the Earth from the satellite at short intervals of Greenwich Mean Time. Then the satellite altitude, or distance in nautical miles from the observer's geographic position, given in an adjacent column, would complete the three-dimensional fix. However, for space navigation near the Earth, and especially for reentry, it will be important that the astronaut know the longitude of his subsatellite position in addition to this information. Rather than supply him with the values of Greenwich hour angle of

Aries at each instant so that he might compute his longitude, one can just as easily give him his longitude directly. This could form an auxiliary column of information in the Space Almanac.

It has been pointed out that the first space penetrations will be near the Earth. In such low orbits, satellite kinetic energy, and therefore velocity, is relatively high, as we have seen in chapter 3. Thus the angular coordinates of the satellite, or conversely, of the Earth as seen from the satellite, will change rapidly. We may draw two important conclusions from this fact. First, the Space Almanac must give positions at very short intervals of time, perhaps every minute, to be of any great value to the astronaut, especially if he is to interpolate between the given times. Second, since the celestial bodies appear to be in very rapid motion when judged by Earth-bound standards, the astronaut needs to have navigational methods with exceptional speed, in order to detect departures of his space vehicle from the predicted orbit.

Heretofore, it has been customary to compute the selected satellite orbit by celestial mechanics, taking into account all the known forces and perturbations which may affect the vehicle. This is possible for unmanned satellites, since the intricate calculations may be done at leisure on the ground before the launch, and improved afterwards from observational data. It will not be feasible for the space navigator to do complicated problems in celestial mechanics in determining his own orbit. Requiring the astronaut to compute his orbit in space would be placing a tremendous burden upon him. Yet for the sake of reliability, he must be provided with an alternative system or method of space navigation, in case the automatic guidance equipment should fail. Whether we call this a primary, a secondary, or an emergency guidance system is of little importance. The fact remains that such a system must be provided.

The heart of such an alternative system could be a well-planned Space Almanac. The information suggested above for inclusion in the almanac—declination and sidereal hour angle of the Earth as seen from the satellite, terrestrial longitude, and altitude above the geographic position, will not enable the astronaut to directly determine his orbital elements when the observed data departs from the predicted data, but only to notice quickly that such a departure has happened. Since his methods must be rapid, the astronaut should have a very efficient method of determining the Earth's apparent coordinates. In the final analysis of the methods of chapter 7, we see that, to obtain accurate positional data, the space navigator must measure the angle between the Earth's center and some known stars. Rather than ask him to then compute the Earth's coordinates from this information, which is a very tedious analytic process, and rather than have him lose accuracy in approximate graphical methods, we can also precompute the values of the angles between various selected stars and the Earth's rim at each of the times given. In addition to giving the altitude of the satellite in nautical miles for information purposes, the almanac could also give the subtended angle of the Earth's disk, thus eliminating the need for referring to another table, such as figure 705A.

With careful planning, the Space Almanac should not only list the angles between the Earth's rim and several conveniently located navigational stars, but also might indicate how slight discrepancies in the observed angles at each of the given times would reflect certain changes in the orbital elements of the satellite. It might be, for example, that under certain conditions the astronaut should turn to another page of the almanac, to pick up an ephemeris closer to his actual orbit.

904. A POSSIBLE ALMANAC FORMAT

The first Space Almanac will find more use by Earth-bound people than by astronauts in orbit. We therefore propose that our first published *Space Almanac* cater to the needs of teachers and students. One way to help them will be to supplement the ephemeris suggested above with other data, such as the orbital elements, rates of precession, etc. Another is to give them a convenient ephemeris of some visible unmanned satellites, such as Echo I, in terms of the latitude and longitude (or sidereal hour angle) of the satellite's geographic position, height above the Earth, and times in sunlight.

With the support of the U. S. Naval Weapons Laboratory at Dahlgren, Virginia, and the U. S. Naval Observatory, we have attempted to provide a space navigator for Echo satellite with usable data which we hope will grow up to be a *Space Almanac*.

Finally, the proposed page layout for the *Space Almanac* is shown in figures 904A and 904B. The redundant data in each line are in reality answers to problems which may be solved from the basic tabulated data. This use of the proposed almanac should be stressed by teachers in the future space navigation classes.

5 JANUARY 1961

ECHO I FROM EARTH

GMT	LONG	LAT/DEC	SHA	SPEED	REMARKS
0^h00^m	W115.36	S08.46	011.01	14300	
05	104.90	19.99	359.30	14000	
10	93.25	30.23	346.39	13720	
15	79.54	38.62	331.43	13470	
20	63.24	44.50	313.88	13220	
25	44.86	47.17	294.24	13000	
30	26.30	46.36	274.43	12890	
35	9.67	42.44	256.55	12780	
40	E 4.17	36.19	241.45	12740	
45	15.53	28.33	228.84	12780	
5[illegible]	[illegible]5.14	19.40	217.98	12890	
		09.72	208.11	13040	
		[illegible]9	198.58	13250	

Figure 904A.–Proposed page layout for a Space Almanac (Left-hand page).

The layout, while prepared for the ultimate use of the space navigator, caters to the needs of the instructor and students a few years ahead of the astronaut himself. This is based on the assumption that several years will be required to put any considerable number of space men in orbit. The plan is to give for selected orbits sufficient data for use in the classroom as well as for use in space.

In addition to the page layout for the Earth ephemeris, several hundred selected star positions should be included for reference. Convenient small-scale star charts could also be included in the *Space Almanac,* although it is possible that the space navigator will be equipped with large, accurately made star charts or star globes.

	STAR DISTANCES		EARTH FROM ECHO I			
GMT	ANTARES	SIRIUS	DISK	RANGE	DEC	SHA
0^h00^m	26.19	18.59	113.23	684	N08.46	191.01
05	24.25	31.55	109.70	769	19.99	179.30
10	22.50	47.30	106.60	858	30.23	166.39
15	22.10	ALTAIR 32.50	103.60	944	38.62	151.43
20	22.69	19.69	101.[illegible]	1022	44.50	133.88
25					47.17	114.24
					36	094.43
						76.55

Anomalistic Period P = 117.1837 minutes
Semi-major Axis a = 4283.5 nautical miles
Eccentricity e = 0.07923
Inclination i = 47.273°
SHA of Ascending Node Ω = 205.273°
Argument of Perigee ω = 133.919°
Mean Anomaly M = 263.262°
(at the Epoch of 00 hours 7 January 1961)

Figure 904B.—Proposed page layout for a Space Almanac (Right-hand page).

CHAPTER 10

RENDEZVOUS IN SPACE

1001. RENDEZVOUS WITH THE MOON

Man has been thinking of going to the Moon and the other planets for possibly hundreds of years, as evidenced by Jules Verne's book "From the Earth to the Moon" published in 1865. What might surprise most of us, however, is that one of today's prime references for trajectories in the Earth-Moon space is computations by G. H. Darwin, reported in 1897. Since the dawn of the space age, a great deal of attention has been given to the problems of a flight to the Moon. Aside from the human desire to conquer space, and possible military applications, there are many scientific reasons for going to the Moon. For instance, current estimates of the mass of the Moon may be in error up to 0.3 percent, and at present we have no knowledge whatsoever of the Moon's magnetic field. Also, properties of the lunar surface and atmosphere such as composition, temperature, radioactivity, and electricity are still in question. The unveiling of these mysteries would give us further clues to the characteristics and history of our own Earth, as well as the other planets.

A diagram of the Earth-Moon system is given in figure 200. A typical Moon-rocket trajectory is shown in figure 1001. A vehicle in this transit trajectory will move in a counterclockwise direction in the initial phase of flight, thus utilizing the orbital and rotational motions of the Earth to help build up the required initial velocity of the vehicle. Such an orbit is referred to as a direct orbit.

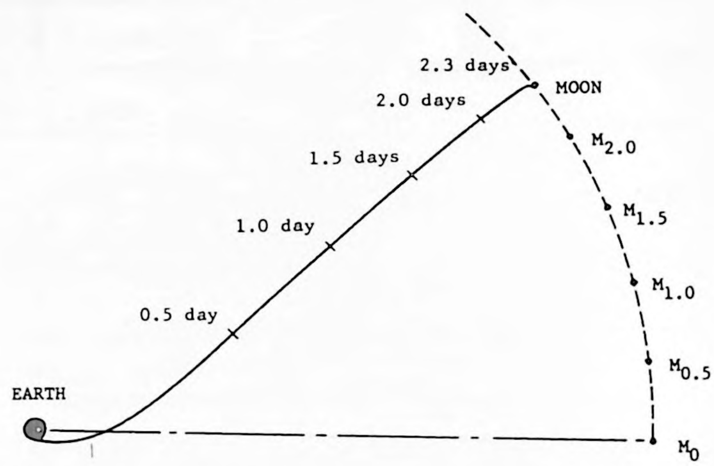

Figure 1001.—A trajectory from the Earth to the Moon. Note the conic-section shape in the vicinity of the Earth, and that the attraction of the Moon becomes apparent only near the end of the trajectory. This trajectory was computed by automatic machines and is taken from the article by H. A. Lieske, "Lunar Instrument Carrier-Trajectory Studies," Rand Corporation, Research Memorandum, RM-1728, June 4, 1956.

1002. INITIAL VELOCITY AND TIME RELATIONSHIP FOR A FLIGHT TO THE MOON

For a flight which does not use substantial velocity changes en route, the time required to reach the Moon is naturally dependent upon the initial velocity. This dependency is particularly critical in the low velocity ranges because of the great distance involved.

Figure 1002 gives the initial velocity in knots versus the flight time in days.

The minimum initial velocity which will enable a vehicle to reach the Moon is 20,562 kt. We can see from the graph that for initial velocities approaching this value, the flight time becomes very large. As previously mentioned, the flight time in the low velocity ranges is critically dependent upon the value of velocity. For example, for an initial velocity of 20,600 kt, the flight time is 5 days. A 1% increase of 206 kt to an initial velocity of 20,806 kt reduces the flight time to 2 days, a 60% reduction in time. A further velocity increase of 1,092 kt would reduce the flight time to 1 day.

The weight of fuel required to produce the chosen initial velocity determines the payload which can be taken along. Therefore, a decrease in flight time is associated with a decrease in payload. However, shorter flight times might allow the required payload to be smaller. In the case of manned flights, for example, less

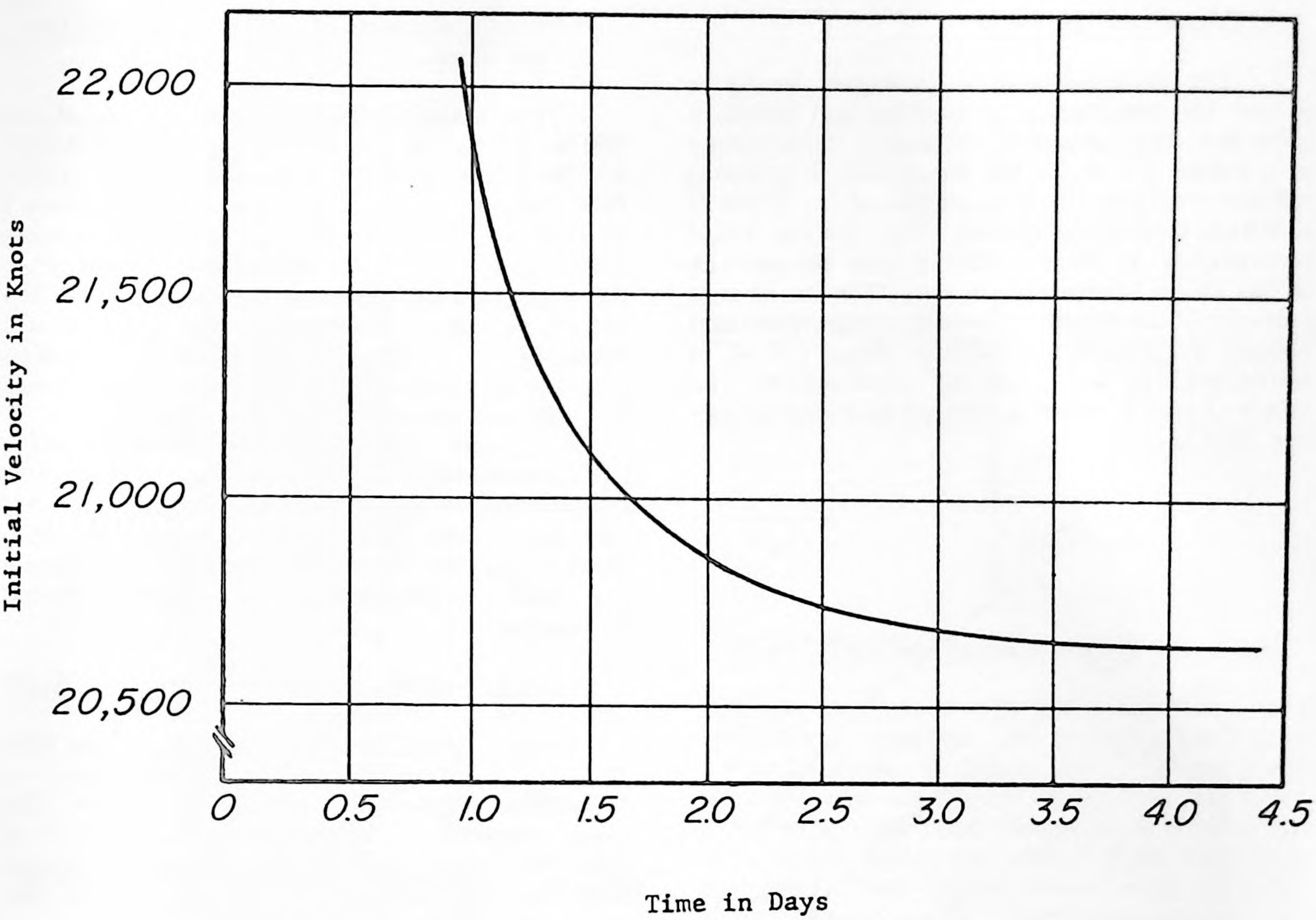

Figure 1002.–Initial velocity vs. flight time. For this graph the initial position of the trajectory is approximately 3,750 N. miles from the Earth's center, or at an altitude of approximately 300 N. miles.

weight in the way of nutrition and livable conditions would be required.

Associated with long flight times is the problem of returning to a given landing site on the Earth. Since small variations in low initial velocities produce such drastic changes in flight time, it would be difficult simply because of the rotation of the Earth to return to a given location with a minimum of powered maneuvers. For example, for an unpowered circumlunar flight, the initial velocity would have to be controlled within about ±0.3 kt for a recovery within the continental United States.[4]

On the other hand, if it is desired to use the Moon's gravitational field to substantially change the course of the vehicle, as in an unpowered flight around the Moon and back to the Earth, then the initial velocity would have to be kept relatively low. An unpowered trip would have a duration of a week or more.

If the vehicle had a maneuvering capability and could commence and terminate the trip at a space station, the advantages of both the long and short flight times could be incorporated into a single trip.

1003. SEVERAL TYPES OF LUNAR RENDEZVOUS

Flights to the Moon would fall into one of the following categories:

1. Impacts on the Moon.
2. Nondestructive landings on the Moon.
3. Establishment of artificial satellites on the Moon.
4. Circumlunar flights, returning to Earth.
5. Lunar passages of escape from the Earth-Moon system.
6. Establishment of libration point buoys.

These will now be discussed briefly. The specific values referred to in this section are taken from *Space Technology*, edited by Howard Seifert.[4]

1. Impact on the Moon.

The main purpose of an impact would be to test the accuracy of projection and guidance systems. The payload would then be flash powder or a bomb. To strike the Moon without powered maneuvers near the termination of the flight is a difficult problem indeed. The vehicle would be traveling at about 6,000 kt near the surface of the Moon (certainly no less than the escape velocity of the Moon). The allowable projection errors at the Earth would be about ±24 kt in initial velocity and about ±15° in direction. The effect of initial velocity errors is shown in figure 1003A.

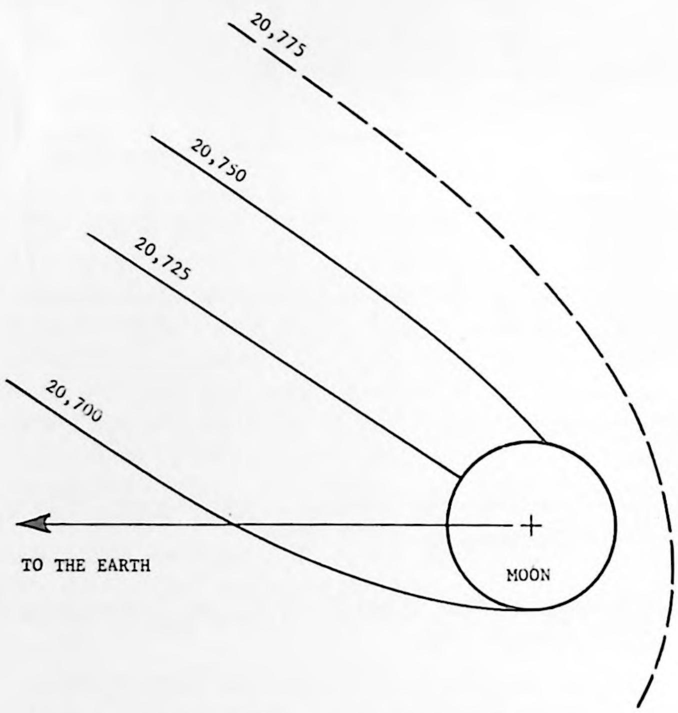

Figure 1003A.–Effect on Impact Point of Varying Initial Velocity.

Another source of error is error on time of launch. The time and the angle of launch are tied closely together because of the Earth's rotation, and so time becomes a critical variable also.

2. Nondestructive landings on the Moon.

A "soft" landing requires the capability of reducing the velocity of the vehicle as it approaches the Moon's surface, which implies the ability of the vehicle to orient itself in space. Also, sophisticated equipment would no doubt be needed to effect a landing and takeoff at the correct time on the surface of the Moon.

3. Establishment of artificial satellites of the Moon.

For a lunar satellite at an altitude of about 200 N. miles, the period would be about 5 hours and the velocity would be about 3,000 kt. Therefore, the velocity of the vehicle would have to be reduced to, or held to, 3,000 kt as the vehicle approached the Moon. The guidance and velocity requirements are not as stringent as they are for a "soft" landing, but the functions performed by the vehicle are similar. The accuracy of projection does not differ significantly from that required for a lunar impact.

A lunar satellite would be an excellent tool for making a more accurate measurement of the Moon's mass (see section 302). A satellite would have to be a mile in diameter to be seen by the naked eye, but one 10 feet in diameter could be seen with the aid of a 40-inch telescope.

4. Circumlunar flights, returning to Earth.

The type of round trip flight to the Moon which is most feasible at the present time is an unpowered flight which circles the Moon. This would require no powered maneuvers at the Moon end, thus saving on payload fuel weight. Also, for safety and payload reasons, the first flight may not be a manned one.

As mentioned previously, an unpowered flight would require a low initial velocity so that the vehicle would be travelling slow enough for the Moon's gravitational field to have the required effect on the trajectory. The well-known "figure eight" trajectory is shown in figure 1003B. It can be seen in the figure that the loop of the figure eight at the Moon end is considerably smaller than at the Earth end. This helps demonstrate why the initial conditions at the beginning of the flight at the Earth are so critical.

The accuracies required for return to the Earth are about ±45 kt in initial velocity or ±5° in direction. For a vehicle with intended initial velocity of 20,700 kt and distance of closest approach to the Moon of 3,500 N. miles, a variation of 6 kt in initial velocity would change the distance of closest approach by about 900 N. miles and the flight time by about one day.

5. Lunar passages of escape from the Earth-Moon system.

Because of the Moon's gravitational field, it is easier to escape from the Earth-Moon system along a trajectory which passes close to

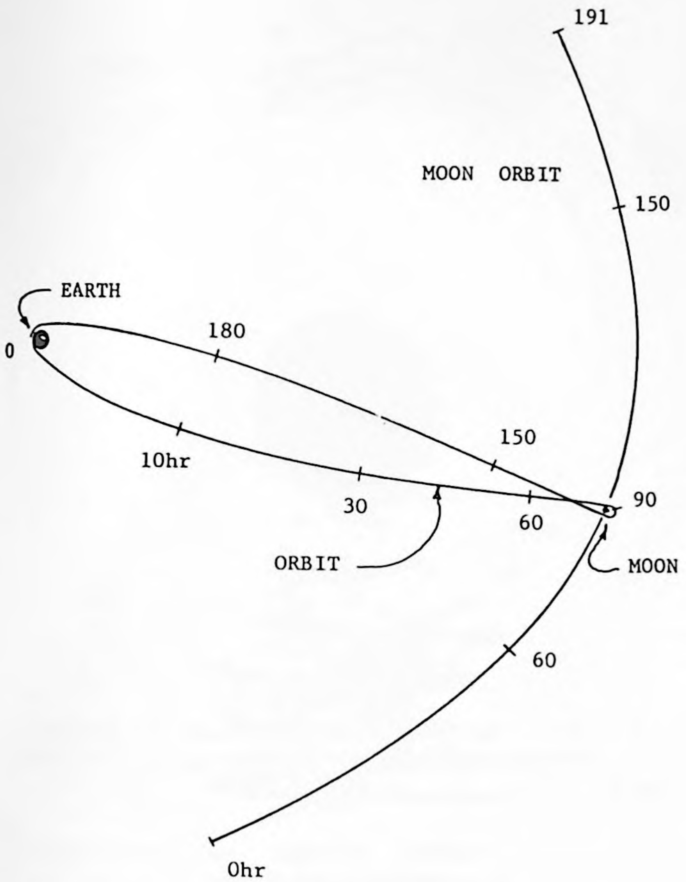

Figure 1003B.–"Figure eight" trajectory for an unpowered flight to the Moon and back.

the Moon. Therefore, travel to another planet might be accomplished by waiting until the Earth, Moon, and target planet were in such a position that a "near miss" shot to the Moon could be used as the initial portion of the flight. Requirements for this trajectory are similar to those for impact.

6. Establishment of libration point buoys.

There is a point in space on the imaginary line joining the Earth and Moon, and approximately where the gravitational field strengths of the two bodies are equal, where a third body could theoretically remain at rest relative to the Earth and Moon. It is intuitively obvious that this is an unstable point; any slight deviation from this location would cause the third body to fall toward either the Earth or the Moon, depending upon the direction of its initial dislocation.

There are, however, two stable locations, or libration points as they are called, which are located at equal distances from the Earth and the Moon such that lines connecting them with the centers of the Earth and Moon, along

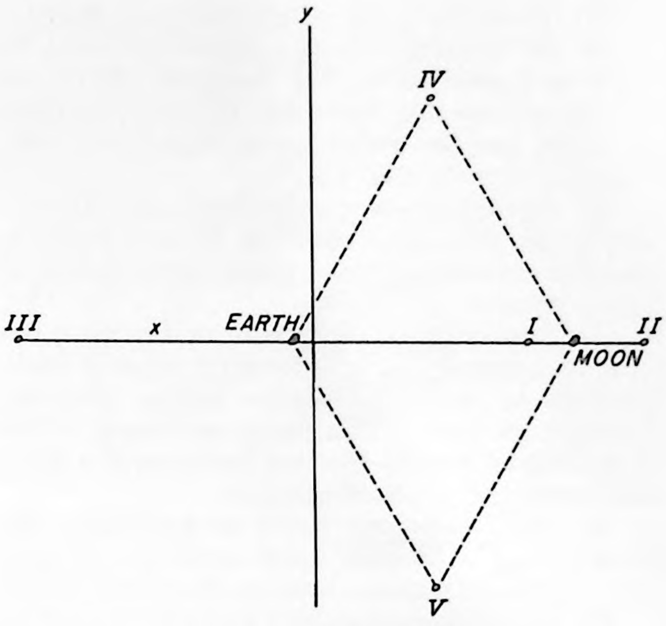

Figure 1003C.–Relation Position of Libration Points.

with the line between the Earth and Moon, form equilateral triangles. The arrangement of these points is shown in figure 1003C. These points IV and V are considered stable because a dislocation of a third body from one of these points results in a stable oscillation of the body about that point. Thus, "space buoys" might be established at these points to serve various functions. Note also that two more unstable libration points exist on the extended line between the Earth and the Moon, one on the Moon side and one on the Earth side.

The determination of the existence of these libration points follows from the solution of the equations of motion of the three-body problem, and is considered to be beyond the scope of this Handbook.

In conclusion, a rendezvous with the Moon is seen to have many practical applications. It will not be too far in the future when navigation to and in the vicinity of the Moon will be an "everyday" necessity.

1004. RENDEZVOUS WITH CLOSE-IN EARTH SATELLITES

The present and future development of large rocket boosters will permit significant increases in satellite payloads delivered into orbit. This in turn will allow maneuverable characteristics of varying degrees to be included in these satellites.

Some possible missions requiring maneuvering capabilities of a satellite are as follows: [2]

(1) Rendezvous for space system assembly, as in the construction of a space station to be used as a manned orbital research laboratory and/or a base for assembly of interplanetary vehicles; and the maintenance, repair, and modification of such stations.

(2) Reconnaissance operations, such as scientific or military inspection of land masses, weather formations, and close surveillance of space objects.

(3) Rescue and retrieve or destruction, the rescue function being necessary when a malfunction in space endangers human life; the retrieve or destruction being necessary when it is desired to stop the transmission of a malfunctioning or outdated satellite.

(4) Station-keeping would be desirable, for example, in a 24-hour orbit used for navigation, communications, and/or reconnaissance.

(5) Deorbiting versatility would be increased were the satellite able to maneuver prior to the initiation of the deorbiting thrusts.

Only several of these missions require the rendezvous of satellites, but the underlying problem is the same: getting to a specified position in space at a specified time with a specified velocity. A rendezvous in space would be accomplished by a succession of finer and finer powered maneuvers interspersed by coasting periods. The actual number of thrust periods will vary according to the problem at hand, but the rendezvous flight is broken into three maneuver phases which are termed: (1) "gross" or initial, (2) intermediate, and (3) terminal, illustrated in figure 1004A. The thrust, guidance, and control requirements vary during the rendezvous flight so the division into three phases not only aids in discussion, but helps clarify the difference of purpose of the various thrust periods.

For a vehicle which starts from the Earth's surface to rendezvous with a nonmaneuvering satellite, the gross maneuver launches the vehicle and imparts a velocity which will enable it to coast to a point in space near the target. At this point, the vehicle does not have enough velocity to go into orbit and, unless a second burst was applied, would follow a ballistic trajectory back to the Earth. At this point the intermediate maneuver, which may be a single thrust or a series of thrusts, imparts the target's velocity to the rendezvous vehicle and places it on a closing course with the target.

The terminal maneuvers then refine the final approach course and velocity, and bring the vehicle to zero relative velocity when the target is reached, or set up conditions which will bring

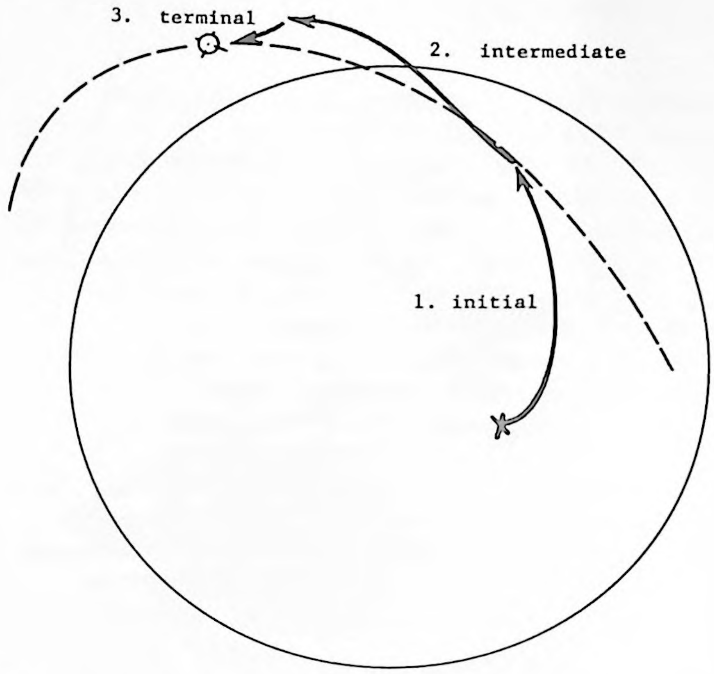

Figure 1004A.—The three maneuver phases for rendezvous with a close-in satellite from a starting point on the Earth's surface.

the vehicle within a certain range at a certain velocity, if this is the desired end.

The gross and intermediate maneuvers can be controlled by inertial means, whereas data of relative position and velocity of the target, measured by sensors on board the rendezvous vehicle, will be necessary for the terminal maneuver.

Figures 1004B, 1004C, and 1004D, show several types of maneuvers which a satellite already in orbit might use to effect a rendezvous. To increase or decrease the radius of the orbit, either an impulsive or a continuous thrust can be used, as illustrated in figures 1004B and 1004C, respectively. To make the outer orbit circular, in the case where the vehicle is maneuvering to increase its orbital radius, an appropriate thrust must be applied at the end of the maneuver or an elliptical orbit will result. The dotted line in figure 1004B represents the trajectory which the vehicle would follow were the final thrust not applied. The dotted line in figure 1004C represents the trajectory for continuous thrust, were the final thrust not applied.

A rendezvous flight may fall into one of several categories. In the case of regular replenishment of a space station for example, where the future positions of the target are known with high accuracy, an unmanned supply vehicle might be launched from the Earth well in advance of the scheduled arrival time at the space station. Low thrust engines could boost the vehicle over a minimum energy trajectory for

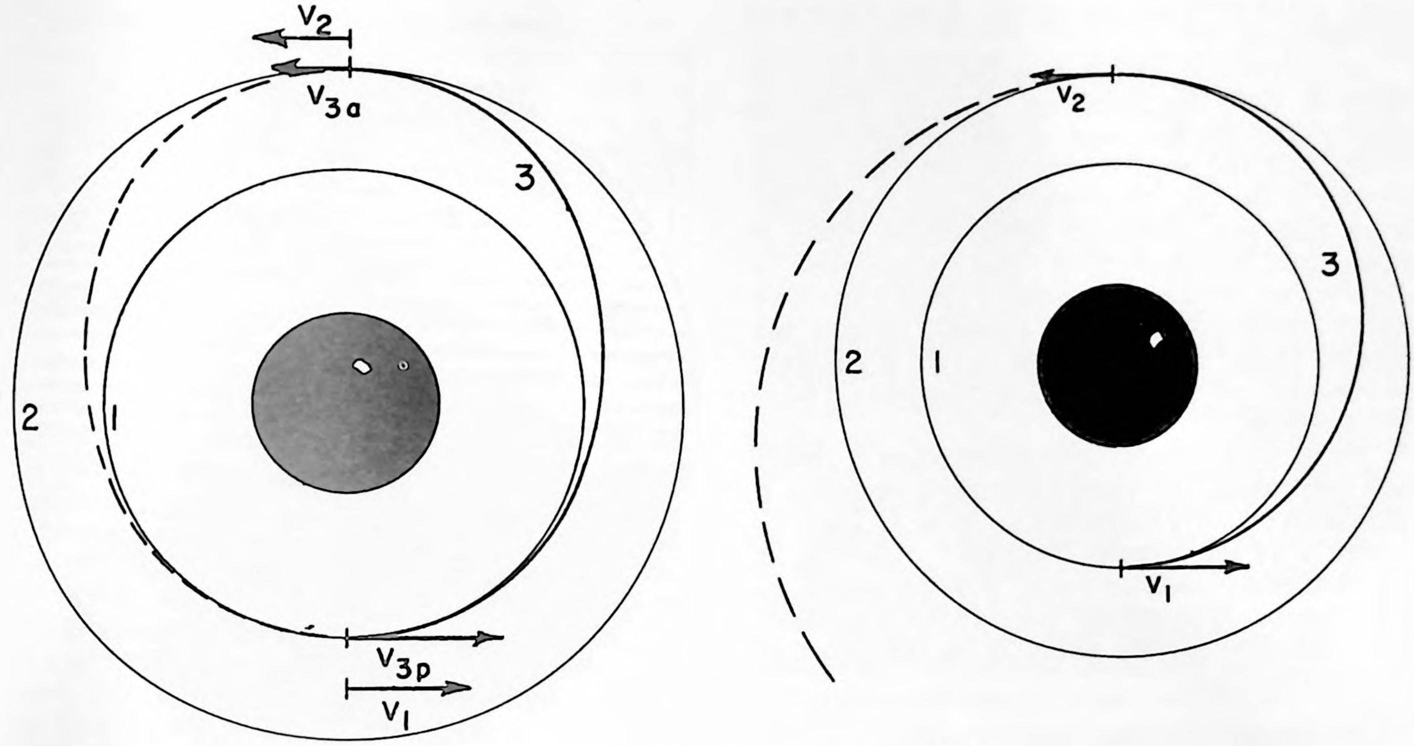

Figure 1004B, 1004C.—Increasing the orbit radius by (B) impulsive thrust, and (C) continuous thrust.

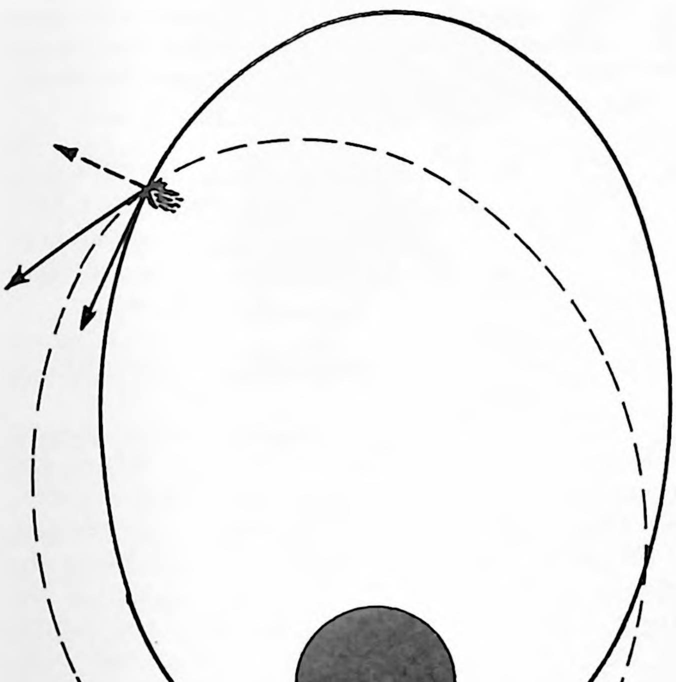

Figure 1004D.—Rotation of the orbit in the orbit plane. If an appropriate maneuver thrust is applied, the axis of the orbital ellipse can be rotated in the orbit plane.

an extended period of time. The efficiency of the low thrust engines would allow greater payloads to be delivered.

If time is an important consideration, high thrust engines would have to be used. In this case, it would be advantageous to wait until the target satellite is in a position which minimizes the rendezvous maneuver and thrust requirements. If the "wait time" between efficient launchings (or orbit changes, if the rendezvous vehicle is already in orbit) is too long for the situation at hand, the rocket would have to effect a more or less severe rotation of its orbit plane (or change in direction of its trajectory).

1005. CHANGING THE ORIENTATION OF THE ORBIT PLANE

An orbiting satellite acts in a manner analogous to a gyroscope: the direction of its angular momentum about the center of the Earth will remain fixed unless a torque is applied. The thrust of the maneuvering vehicle produces a torque which acts through the arm of the radius vector to the center of the Earth. Thus, a thrust applied to the vehicle in a direction perpendicular to its direction of motion and parallel to the Earth's surface does not simply move the vehicle in the direction of the thrust, but rotates the spin plane (orbit plane) about an axis in the

orbit plane through the center of the Earth and the satellite.

An alternate approach to the situation is to observe that the resultant velocity vector is simply the vector sum of the initial velocity vector and the velocity vector imparted by the vehicle's thrust. The new velocity vector defines the new (rotated) orbit plane. This is illustrated in figure 1005. This second point of view makes it easy to see that a small impulsive burst would produce only a small jump in the orbital plane direction. To obtain a constant precession a constant thrust (torque) would have to be applied.

To rotate a circular orbit by the angle θ (without increasing the radial distance) requires a minimum velocity increment equal to $2v \sin \theta/2$, where v is the orbital velocity.[3] Clearly, a large angular rotation would require considerable thrust. (The orbital velocity for a satellite at an altitude of 500 miles is on the order of 15,000 kt.)

1006. GUIDANCE FOR THE THREE RENDEZVOUS PHASES

The guidance of the three phases will now be considered in more detail.

Conditions which are assumed to be present during the rendezvous problem discussed here are:

1. The "target" vehicle is nonmaneuvering.
2. Target position is predictable from ground stations.
3. Thrust periods will be interspersed by coast periods during which thrust commands for the next maneuver may be computed.

(1) Gross Maneuver Guidance.

If the rendezvous vehicle is to be launched from the Earth, it may be possible for the vehicle to be launched directly into the orbit plane of the target vehicle. If the rendezvous vehicle is already in orbit, the gross maneuver would probably be directed toward getting the rendezvous vehicle in the vicinity of the target vehicle, leaving the precise orientation of the orbit plane to the second, intermediate maneuver.

As for actual guidance, this could be accomplished by a computer system which would compare (1) the vehicle's velocity vector with (2) the required velocity vector for rendezvous, to obtain (3) a velocity-to-be-gained vector which would be used for steering and thrust control. The vehicle's velocity vector could be obtained from the computer of an inertial system, and the required velocity vector for rendezvous could be computed using information of the target satellite's behavior obtained from ground stations.

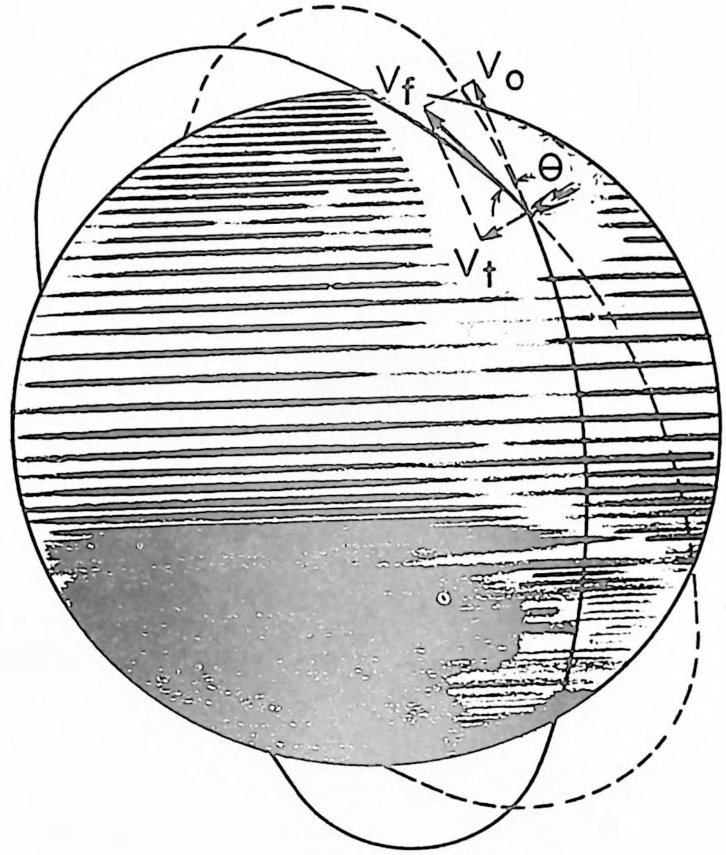

Figure 1005.—Changing the orientation of the orbit plane. The direction of the new orbit plane is determined by the vector sum, v_f, of the original velocity, v_o, and the velocity produced by the vehicle's thrust, v_t.

(2) Guidance for Intermediate Maneuver.

The purpose of the intermediate maneuver is to put the vehicle in a situation where the target will be detected by the vehicle's sensors, and which will allow the terminal corrections to be performed efficiently, i.e., to place the vehicle in an orbit nearly matching that of the target. With the vehicle now near the target, the main requirement of the intermediate maneuver will be to give the maneuvering vehicle the velocity of the target. As previously mentioned, this may involve a more or less severe plane change.

If an "ideal" rocket is assumed, then the time to start thrust, the time to stop, and the direction of thrust for the intermediate maneuver can all be precomputed in the rendezvous vehicle computer during the gross maneuver "coast" phase, using data obtained from the

accelerometers in the gross maneuver "fire" phase. In an actual rocket, where the thrust may not be constant nor the thrust cutoff precise, a new velocity-to-be-gained vector may have to be continuously computed during the intermediate "fire" period. This continuously changing vector would then be used for steering and cutoff.

(3) Guidance in the Terminal Correction.

At the onset of the terminal phase, that is, when the relative position sensors have locked onto the target, the relative closing velocity will probably be of the order of several hundred knots, and the vehicle may not be on a collision course with the target. So, the initial thrust will adjust the closing velocity and put the vehicle on a collision course, or on a course which will give the desired CPA (closest point of approach). The last thrust reduces the relative velocity to zero or a predetermined value as the relative range reaches zero or nears the desired CPA (closest point of approach). Thus at least two thrust periods are required in the terminal correction, and extra thrusts between these two may be used to refine the collision course, reduce the closing velocity as range decreases, and/or bring the vehicle in to the target from some preferred direction.

The sensor system may use either radar, infrared, or optical devices. The equipment and power required for a radar system will depend upon the extent to which cooperative devices on the two satellites can be used. The three possible situations are as follows:

1. The rendezvous vehicle carries all of the sensory equipment, the target being used only as a reflector.

2. Both satellites carry sensor equipment and are actively engaged in the terminal guidance.

3. The target satellite carries all of the sensor equipment and transmits the required data to the maneuvering satellite.

In case 1, the power needed for a conventional radar system would depend on the detection range demanded for a target of a given size. For a range of 20 miles, a power on the order of 100 kilowatts would be expected.[2] The power, however, increases as the fourth power of the range.

In case 2, the maneuvering vehicle might monitor a constant frequency signal transmitted from the target to determine relative velocity using the Doppler effect. (The Doppler technique might also be practical in determining the relative velocity in case 1.) Range could be found by triangulation methods or by noting the time delay between the signal received directly from the target and the one reflected from the earth. Or, a signal from the maneuvering satellite might activate a transponder in the target which would retransmit the signal, from which the maneuvering vehicle could determine range and direction. (This system is reminiscent of the IFF systems used in airplanes.) This arrangement would extend the range to hundreds or possibly thousands of miles at moderate power levels. Ranges of about 50 miles could be obtained with a few watts.[2]

Case 3 is the most efficient for the maneuvering vehicle but naturally makes heavy demands on the target, especially if large ranges are required. A system of this type might be feasible for the assembly or maintenance of large space stations.

Optical methods may be practical for the last phases of the terminal maneuver, for they offer a very simple and accurate method of obtaining line of sight information. The predicted range of 150 miles in sunlight could be extended to 2,000 miles if a 4-inch telescope were employed.[2] Power requirements for optical detection in the absence of sunlight would be heavy, but would be comparable with one-sided radar systems. If the terminal sensor system could be instrumented practicably to give a velocity-to-be-gained vector, then the vehicle's inertial system could be used to control the vehicle during the terminal corrections as it did during the first two maneuvers. This might be especially useful in the longer range terminal corrections. Also, it may be necessary or practical to utilize some sort of grappling device to bring the two satellites into direct contact.

Recent advances in microminiaturization suggest that it will be possible to mechanize very complex systems with equipment of extremely small size and weight. However, the systems used will no doubt be a compromise between the complexity required for optimum performance and the simplicity desired for good reliability.

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CHAPTER 11

REENTRY

A most important consideration in the flights of manned space vehicles is that of reentry. Recovery of the human passengers unharmed is of paramount importance. As we have seen in chapter 3, the spacecraft receives a large quantity of energy when it is put into orbit. This total energy exists in two forms, kinetic energy of motion and potential energy due to position in the Earth's gravitational field. The total energy of the vehicle is one of the independent parameters that help define its orbit.

Before the vehicle can return to the surface of the Earth, this energy must in some way be dissipated. This defines the reentry problem. Techniques for effecting reentry must also reflect questions of navigation and aerodynamic effects. It is important that reentry allow control of the flight path, so that the vehicle may be guided to a desired spot on the Earth. Heating, deceleration, and other considerations must also be included in the treatment of the reentry problem.

1101. ATMOSPHERIC RETARDATION

Although the velocities of space vehicles are very high by terrestrial standards (22,000 knots for a lunar flight), the problem may be treated nonrelativistically. The kinetic energy is then given by the classical equation $T = (1/2)\ m v^2$, where m is the mass of the vehicle and v the magnitude of its velocity. The specific energy, the energy per unit mass, is independent of the mass of the satellite. Thus, the kinetic (and total) specific energy of the vehicle may be reduced by reducing the velocity. Two methods come immediately to mind.

Braking (retrograde) rockets could decelerate the space vehicle in precisely the same way that the launch rockets accelerate it. Unfortunately, with the chemical power plants now used, the retrorockets would need to be large, and great amounts of fuel would be required to slow the craft. In addition, the large rockets and their fuel would have to be launched into orbit with the actual payload. Even when very powerful boosters become available which are capable of launching tremendous weights, the use of braking rockets as a primary reentry system would be unattractive. Yet present spacecraft must depend on chemical rockets as their only means of control. The inefficiency of this technique causes us to look in other directions for a more desirable solution to the reentry problem.

A second method would take advantage of the retarding effects of the Earth's atmosphere. As a body moves through the air, a strong shock wave is formed in front of it. The kinetic energy of the moving vehicle is converted into thermal energy, as the air molecules are heated and dissociation takes place. Thus, the atmosphere serves as a braking instrument for the reentering craft. Since this method has no additional propulsion requirements, except perhaps for control, it seems to be much the better of the two. But there are also problems inherent to this type of reentry.

1102. AERODYNAMIC HEATING

As the vehicle moves through the air, it transfers a portion of its kinetic energy to the air as a result of the collisions of the surface of the vehicle with the stationary air molecules. At the hypersonic velocities a reentry vehicle experiences, an important effect takes place. Part of the air begins to move along with the aerodynamic surface, forming a viscous layer. The air in the viscous layer is at very high temperature, and it heats the vehicle by conduction and convection.

Another effect also becomes important at high velocities. The air in front of the vehicle is compressed extremely rapidly in a strong shock wave. The final result is a dissociation and ionization of the molecules; this, however, takes a finite time to happen. The air at the very front of the shock wave is hence heated to extremely high temperatures. By radiation it in turn heats the vehicle, this effect increasing with the square of the velocity. At about 15,000

knots it is smaller than the viscous layer heating; about 21,000 knots, the heating rates are nearly equal; and above 27,000 knots this effect predominates. In a properly designed aerodynamic vehicle only 1% or 2% of the total kinetic energy is converted to the heating of the vehicle. The rest goes into dissociation and ionization of the air and into heating the air, primarily through the formation of the shock wave.

The rate at which a vehicle is heated upon reentry depends on its original velocity and on the reentry path. For a given velocity, heating rates are higher at lower altitudes, as one would expect, since the atmosphere is denser at lower levels. It is therefore advantageous for the vehicle to decelerate as high in the atmosphere as possible.

If the aerodynamic heating rate becomes too high, it exceeds the rate at which the vehicle can dissipate heat, and the vehicle burns up, much as do the many meteorites entering our atmosphere. There are two principal ways to dissipate the heat, radiation and ablation. Radiation cooling is used primarily with lifting space gliders, which enter the atmosphere at a shallow angle. The ablative process, in which a nonmetallic heat sink of high specific heat capacity is allowed to char and burn up, is employed with steeper reentry ballistic vehicles. Other methods, such as the use of liquid cooling systems, have also been advanced. Maximum heating rates tolerable depend on the design of the vehicle and the materials of which it is made.

1103. DECELERATION

Another critical factor in reentry is deceleration. Physiological tests have shown that humans, with properly designed equipment, can perform control functions almost normally while undergoing sustained decelerations of eight to ten G's (i.e., eight to ten times the acceleration due to the earth's gravity). A human can survive as much as twenty G's, but only for a very short period. The ballistic reentry must then be such as not to exceed tolerable deceleration forces. It is interesting to note that maximum deceleration does not depend upon the height at which it occurs. Thus deceleration high in the atmosphere will produce approximately the same forces as will deceleration at lower altitudes.

1104. REENTRY FROM A CLOSE CIRCULAR ORBIT

In pursuing the reentry problem, we shall consider the two extremal cases — reentry from a near-Earth circular orbit and reentry from an orbit whose apogee point lies in the vicinity of the Moon. Techniques for reentry from intermediate orbits will range between the techniques for the two extremums.

We first take up reentry from a close circular orbit. A prerequisite for reentry initiation is the accurate determination of the orbital parameters. This might well be accomplished by the methods already described in chapters 7 and 8. Since the vehicle velocity is fixed and can be easily found from the orbital parameters, the only variable is the reentry angle. (The reentry angle ϕ is the angle between the vehicle's velocity vector and the horizontal. See figure 1104A.) For a given orbit a precomputed table would provide information on attitude requirements and the magnitude of impulse needed from the retrorockets. Such a table is feasible, since the actual orbit is normally quite close to the planned orbit. The tables would include a small range of eccentricities and major semidiameters. Instead of a tabular display, a simple slide rule computer or nomogram might be devised to supply this information. The reentry angle and the velocity uniquely determine the reentry path. The range from the point where reentry is initiated to the point where the vehicle lands on the earth's surface is most conveniently given by determining the central angle η formed by rays through the Earth's center passing through the two points. This angle, called the range angle, is shown in figure 1004A. An approximate analytic expression for the range angle is

$$\tan \eta/2 = \frac{\sin \phi \cos \phi}{(2g\ R_e/v^2)} - \cos^2 \phi \qquad (1)$$

where ϕ is the reentry angle, R_e is the height (measured from the earth's center) at which reentry is initiated (see figure 1104B), and v is the initial velocity.

A nomogram, such as the one found in the *Missile Engineering Handbook*,[1] can give a quick graphical solution to this equation. According to Robinson and Besonis,[2] a 20-foot per second error in velocity, a 0.1° error in reentry angle, and a 30% uncertainty in the composition of the atmosphere would give a cumulative miss distance of 40 miles for a reentry angle of 2.5°, with diminishing error for increasing range angle. The argument errors are in the expected range, and the total miss distance is certainly tolerable, especially for a ballistic reentry.

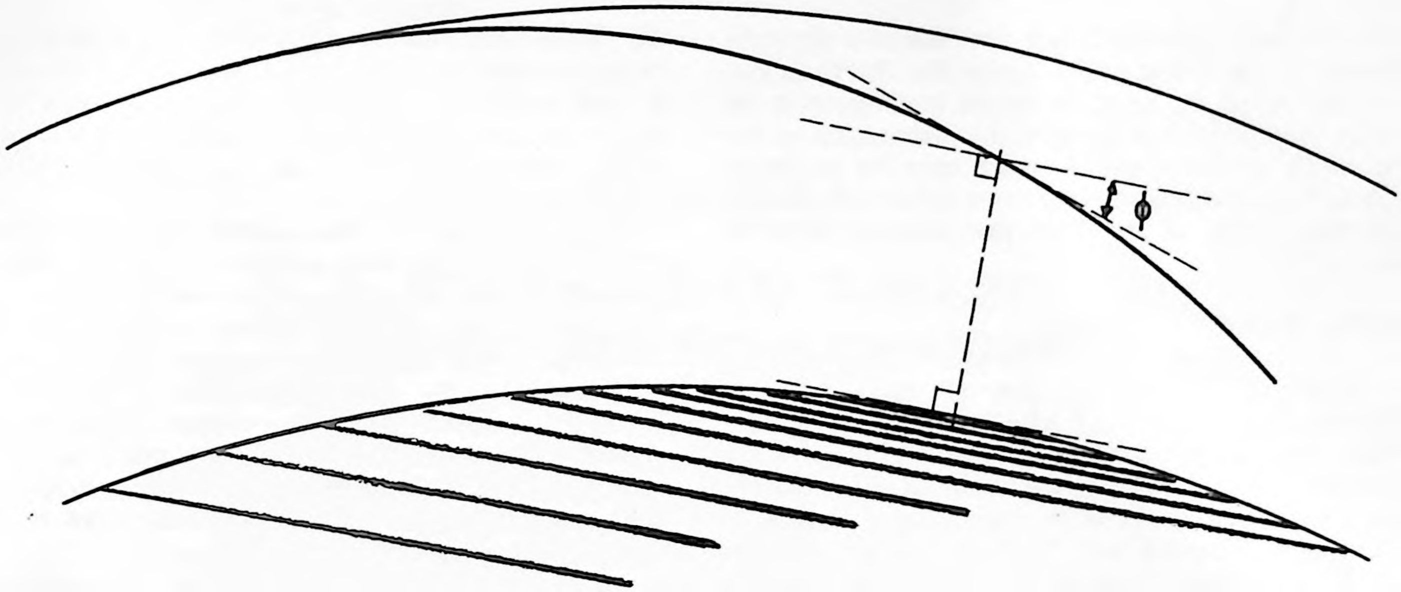

Figure 1104A.—The reentry angle ϕ is the angle between the horizontal and the flight path.

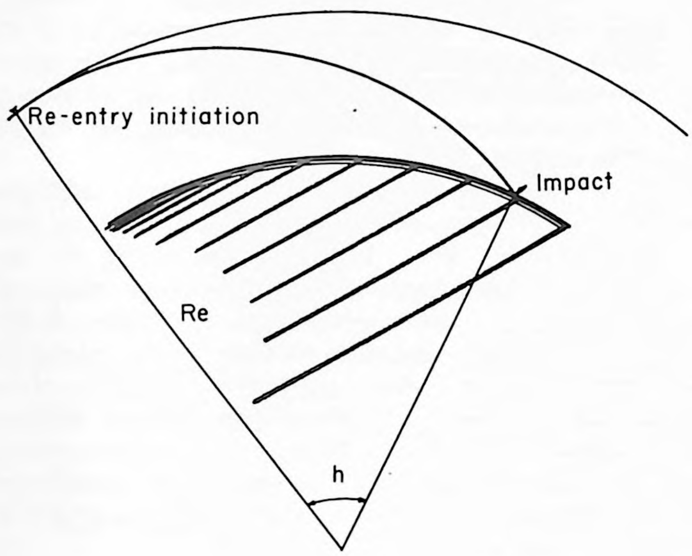

Figure 1104B.—The range angle η and the height of reentry initiation R_e.

1105. REENTRY FROM A LUNAR ORBIT

We next consider reentry from an orbit whose apogee is near the Moon (i.e., about 200,000 N. miles from the Earth). The multiple-entry or braking ellipse method has been advanced as a possible technique. In this method the vehicle passes through the fringes of the atmosphere at perigee, slowing down gradually each pass. The apogee of each succeeding ellipse moves closer to the Earth. This technique was doomed, however, by the discovery by Dr. Van Allen of belts of high energy radiation around the Earth. Multiple entry would repeatedly send the vehicle through the Van Allen belts, subjecting passengers to lethal doses of radiation. It is thus necessary to effect reentry in a single pass, or at least to prevent excessive exposure to radiation.

Reentry on a single pass develops into a critical control problem. At perigee should the vehicle be too far from the Earth, it will skip through the atmosphere and move off into another orbit. Should it be too close, heating rates will be excessive, and the vehicle will burn up. This leaves a very narrow band or "reentry corridor" which the vehicle must hit. For the nonlifting vehicle which we are considering, the band is only eight miles wide.[3]

Such a small target is the reentry corridor that almost impossible accuracy is necessary for an uncorrected approach. If positions can be determined optically and the orbital elements calculated, the height of perigee can be predicted. At various intervals during the approach, corrections must be applied to the flight path. Corrections could be made to keep the perigee merely within the corridor, or, more restrictively, to keep the perigee at the center of the corridor. White, Foudriat, and Young[4] indicate that the latter method, although requiring corrections sooner, will result in a total fuel savings, since corrections near the end of the approach would be smaller.

Even with a single pass reentry possible, the question of accuracy raises an obstacle. It is extremely difficult to predict beforehand the exact time that reentry would commence, due

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to the long approach and the various corrections to the flight path. Since the Earth is rotating about its axis, it would consequently be nigh impossible to predict its orientation at the time of reentry, and it would only be sheerest coincidence if the vehicle were to hit its intended landing area, at least at the present state of the art.

1106. PARKING ORBITS

A second technique, that of using a near-Earth "parking orbit," alleviates this problem. The vehicle would be controlled essentially as before to hit the desired perigee height. Rather than allowing reentry to commence, however, the vehicle would be given a small boost into its close circular parking orbit. The vehicle would coast in this orbit until the Earth were properly oriented for it to make its descent. The problem then degenerates into reentry from a close orbit, which has already been described in section 1104. Although more expensive from a standpoint of fuel consumption, the use of a parking orbit greatly simplifies the problem and increases accuracy immeasurably.

Reentry from orbits between these two extremes can be accomplished by a suitable combination of the methods already described.

1107. MECHANICAL SYSTEMS

Mechanical computing devices can greatly facilitate reentry control for the astronaut. Such instruments were essential on the mercury capsules, our first manned space vehicles. Called an "Earth-path indicator" by its designer, Minneapolis-Honeywell Regulator, it showed the vehicle's instantaneous position on a small revolving globe. The globe was marked with the continents, topography, and major cities. It also showed meridians and parallels of latitude. The globe was driven by a system of friction rollers and gearing.

The system was set before launching for the predicted orbital inclination and period. After launch the astronaut would adjust the settings for these two orbital elements, once they had been accurately established. To adjust the displayed position, the operator could slew the globe to correspond to the observed actual position. From this point on, the indicator operated automatically.

The globe motion was controlled by two drive mechanisms. The first revolved the globe about its polar axis, simulating the actual rotation of the Earth. The second revolved the globe in the plane of the orbit, at an angular velocity corresponding to the satellite's period. The drive mechanism also took into account the major perturbations of the orbit, essentially the precession of the orbital plane. Driven by a spring clock unit, the Earth-path indicator was independent of outside power supplies.

As designed and used in the mercury system, the indicator worked only for circular orbits. In a modified form the system could be made usable for elliptic orbits also. This would involve an additional gear drive which would introduce a cyclic variation in the rotation in the orbital plane. A third input, eccentricity, would modulate this variation. It would also be necessary to compensate for the rotation of the ellipse in the orbital plane.

The viewing face of the mercury indicator showed the vehicle's geographic position, with a set of concentric range circles about it. Another marker indicated the point at which the vehicle would land were reentry initiated at that moment. This marker was at a range along the orbital path corresponding to the altitude of the capsule (which was accurately known beforehand) and the amount of fuel in the retrorockets. For a general case the range angle would need to be computed, and the landing point might be quickly spotted on the globe.

In our quick review of the reentry problem, we have seen that it is indeed a critical one. Deceleration, heating, and accuracy must all be considered. But it is nice to know that, without relying on complicated communication links to the Earth, the astronaut could still safely land his vehicle. The task of reentry, albeit a difficult one, is definitely manageable.

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APPENDIX A

U.S. PATENT NO. 3,002,278
METHOD FOR SPACE NAVIGATION

This patent was issued as this book neared completion. Since it covers the concept on which the book is based, it is reproduced as an Appendix. If history repeats, this will be only one of a spate of patents soon to appear on Space Navigation. For example, before the end of World War II it was reported that 3,000 inventions had been claimed, and that three billion dollars had been spent on radar alone; and that Hitler, referring to radar, stated: "One scientific invention caused my defeat." In any case, our Government encourages the development of new ideas, and this timely patent is a case in point.

The relation between research, a concept, a patent, and descriptive text is explained by a comparison of this Appendix with the preceding text, particularly section 707. The first step in human achievement is a basic concept, and "what man conceives, man can achieve."

Letters patent require meticulous, repetitive, legal descriptions which protect apparatus, but do not protect ideas. The human brain is a unique "idea machine," yet it is a machine which, by its nature, cannot be patented.

Since the purpose of this Handbook is to give descriptions of selected old ideas, and to stimulate the generation of new ideas, students should be familiar with Government Patent policy. The Patent Section of the Navy Department, at no cost to the inventor, processes a patent in his name, and retains only the Government rights, with commercial rights to the inventor. This arrangement encourages inventions to the mutual benefit of the Navy and of the inventor.

We trust this brief discussion of patent procedure will stimulate the imagination and energy of students at this critical period in human affairs.

Oct. 3, 1961 P. V. H. WEEMS 3,002,278

METHOD FOR SPACE NAVIGATION

Filed March 6, 1959 3 Sheets-Sheet 1

FIG. I.

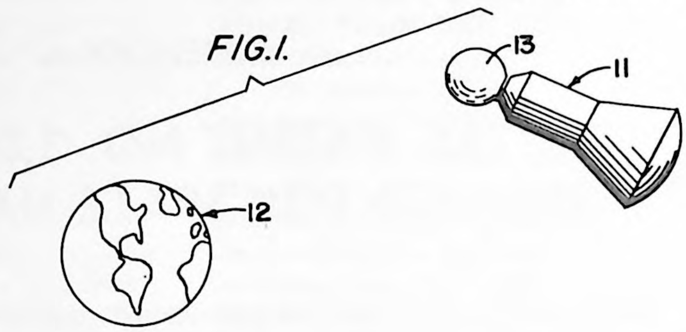
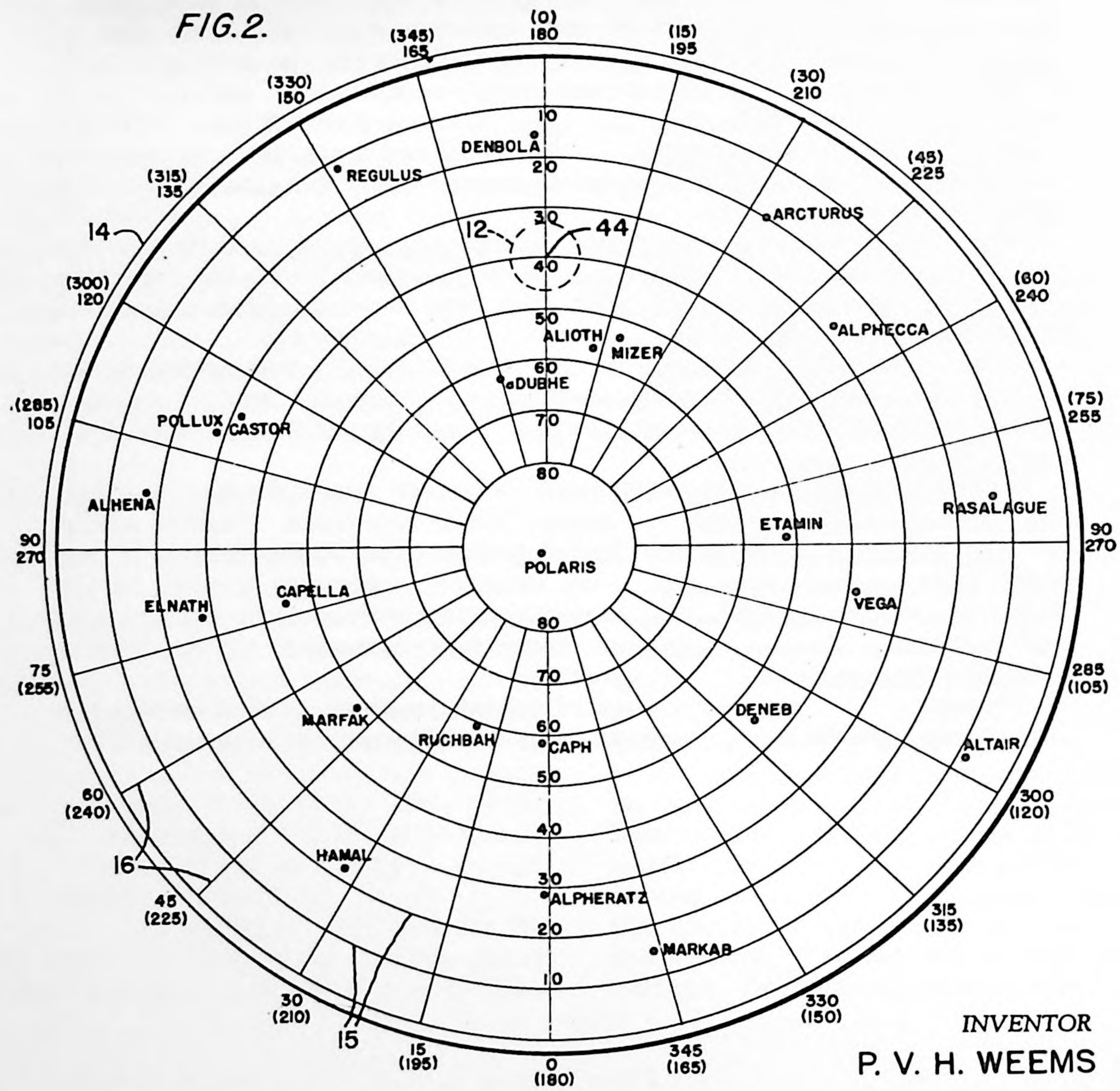


FIG. 2.



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Oct. 3, 1961 P. V. H. WEEMS 3,002,278

METHOD FOR SPACE NAVIGATION

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FIG.3.

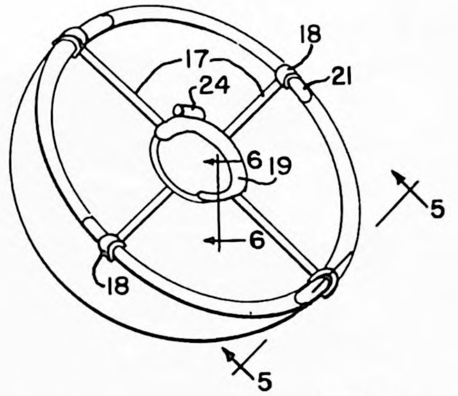

FIG.4.

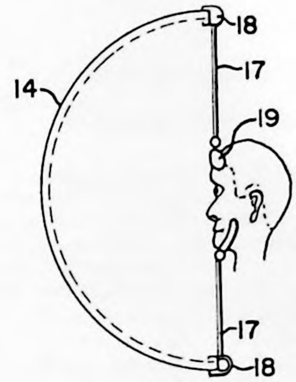

FIG.5.

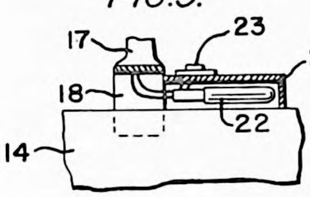

FIG.8.

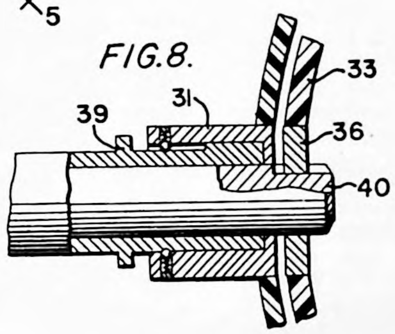

FIG.6.

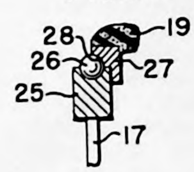

FIG.9.

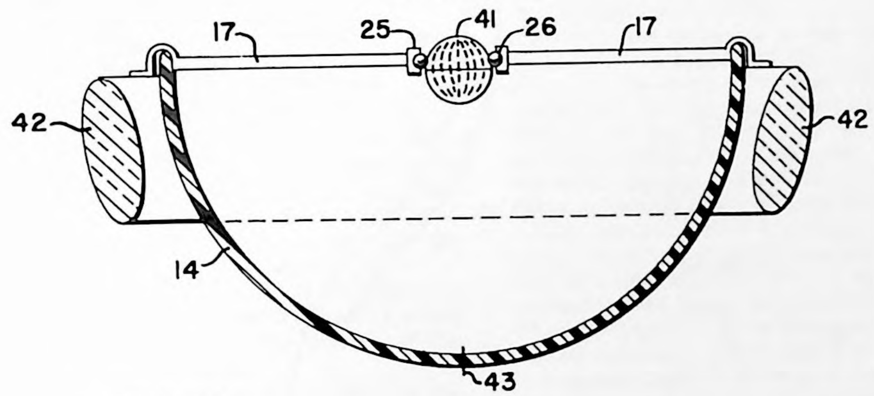

FIG.10.

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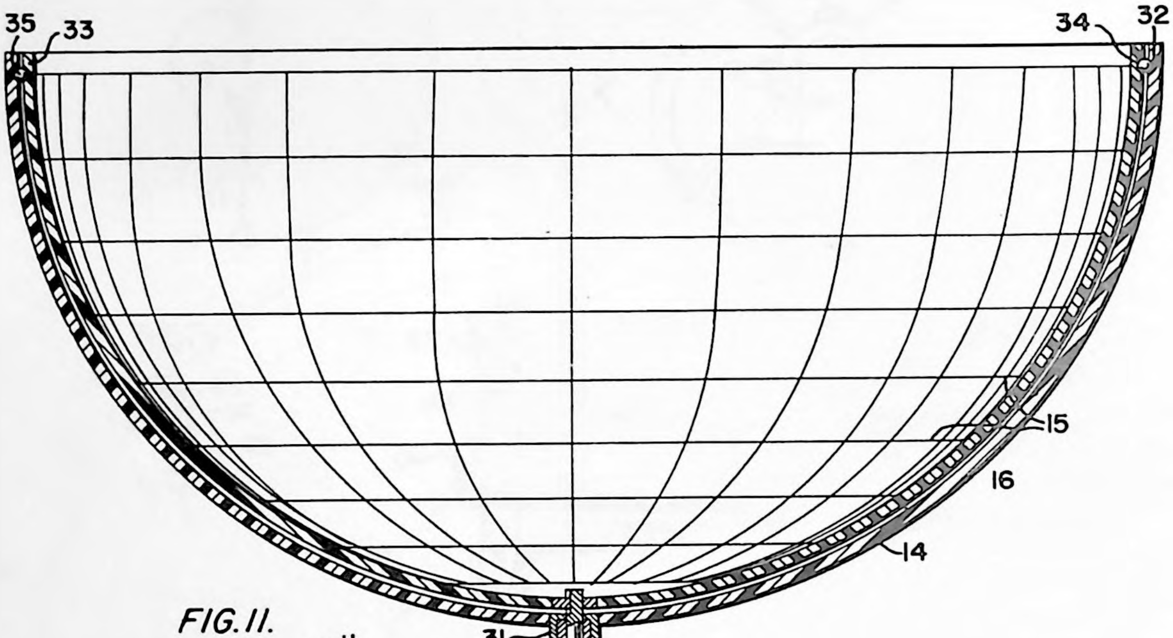
FIG. 7.



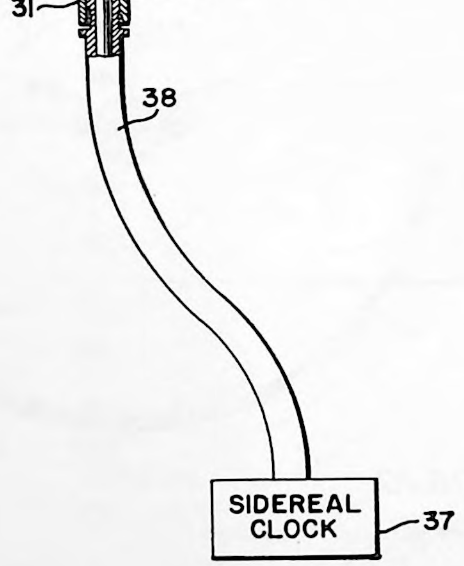
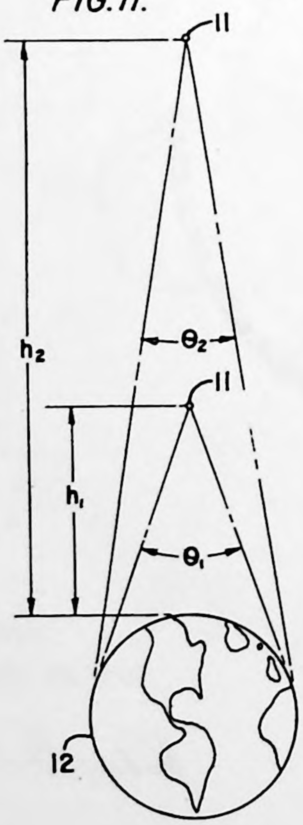


FIG. 11.



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United States Patent Office

3,002,278
Patented Oct. 3, 1961

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3,002,278
METHOD FOR SPACE NAVIGATION
Philip V. H. Weems, Randall House, Annapolis, Md.
Filed Mar. 6, 1959, Ser. No. 797,605
2 Claims. (Cl. 33—1)

This invention relates to space navigation and more particularly to a method which can be employed to determine the position of a vehicle relative to the surface of the earth when it is in orbit or on a trajectory within the relatively near regions of space surrounding earth.

The problem of space navigation has been the subject of considerable study in the recent past and various solutions heretofore proposed are being considered since various plans are well advanced at this time for putting man into orbit and beyond orbit into flight in the interplanetary space of the solar system. As presently conceived, the space flights of the foreseeable future will of necessity be limited to regions which would be considered near the earth in terms of astronomical distance and all such flights will be subject to the imperative condition that means be provided for the occupant of the space vehicle to be returned safely to earth. Accordingly, all such flights as now contemplated will start and terminate on the earth and involve maneuvers in the region approximately between the earth's surface and the orbit of the earth's moon which are crucial to the objective of the flight and particularly to the maneuver by which the occupant will initiate and control the re-entry phase into the earth's atmosphere. While a major portion of the operations performed in such a vehicle may be arranged subject to automatic control, in accordance with program sequences or ground communication link instructions, it is important that the occupant of such a vehicle be able to make determinations of his position in the event of failure of the automatic equipment and in any event for the physiological and psychological benefit which a human being in such circumstances would derive from information presented in terms of a familiar coordinate system.

It is an object of this invention to provide a simple method for navigation in a space vehicle.

A further object of this invention is to provide a simplified method for navigation which permits a direct observation of position relative to the surface of the earth to be made in a space vehicle.

Another object of this invention is to utilize an extremely light weight space navigation device and a method for navigating which is completely devoid of complicated mechanisms and circuits and hence completely immune to the possibility of inoperativeness or malfunction.

These and other objects of the invention will be apparent from the following detailed description taken in conjunction with the accompanying drawings wherein:

FIG. 1 is a representation of a satellite in orbit at a considerable altitude above the surface of the earth;

FIG. 2 is a flat developmental view of a hemispherical navigation device utilizable in accordance with the present invention as viewed from the side of the observer;

FIG. 3 is a perspective view of the navigation device of FIG. 2 showing details of the construction;

FIG. 4 is a sectional side elevational view of the device of FIG. 3;

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FIG. 5 is a view partly in section taken along the line 5—5 of FIG. 3;

FIG. 6 is a sectional view along the line 6—6 of FIG. 5;

FIG. 7 shows a modification of the device for use in accordance with the invention to provide an indication of actual earth latitude and longitude;

FIG. 8 is a sectional view of a drive coupling for the modification shown in FIG. 7;

FIG. 9 shows a modification of the device;

FIG. 10 shows an alternate hemisphere; and

FIG. 11 shows an altitude determination method.

In accordance with one device usable in practicing the present invention a transparent star hemisphere is arranged with a sidereal hour angle and declination grid thereon and a suitable eye piece is provided for sighting from the effective center of the hemisphere out through the hemisphere surface toward the earth disc as observed from the space vehicle. By selecting a suitable hemisphere depending upon the relative position of the earth and the space vehicle, the star field observed beyond the earth and particularly the stars which surround the earth disc as observed from the space vehicle can be aligned with the stars marked on the star globe. Orientation of the star globe such that two known stars on the globe coincide with the observed position of the same stars in the star field surrounding the earth disc will result in orientation of the globe in a manner such that the polar axis of the globe is parallel to the polar axis of the earth. For this condition, the sidereal hour angle and declination grid on the star globe are, in effect, projected onto the celestial sphere in which the star field is observed and the position of the earth disc in this projected grid corresponds with the position of the space vehicle above the surface of the earth. The exact location of the space vehicle will therefore correspond to the center of the earth disc as it is positioned on the sidereal hour angle and declination grid of the star globe. A modification of the device provides a direct indication of the space vehicle's position in terms of earth latitude and longitude.

Referring now to FIG. 1, a schematic representation of a space vehicle **11** in orbit about the earth **12** is shown. The situation depicted in FIG. 1 is representative of a condition which forms an important part of any space travel contemplated in the foreseeable future. Thus, in the near future, the representation of FIG. 1 corresponds to a manned space vehicle in orbit about the earth and at some future date, the launching and re-entry of a space vehicle on more extended flights in the solar system will also include a flight phase corresponding generally to the configuration of FIG. 1. For the purpose of the present invention, a space vehicle **11** is presumed to have a means **13** for visual or optical observation. As represented in FIG. 1 the means **13** comprises a transparent spherical portion of the space vehicle wall which an observer may use to survey the surrounding space over a wide solid angle field of view. Manifestly, any other simple means of observation or optical sighting may be provided in the space vehicle **11** other than as represented.

It is well known in the art of celestial navigation that the earth **12** is positioned in a relatively fixed star field with the positions of the various stars with respect to the center of the earth fixed over long periods of time and that the angular positions thereof are accurately known. The rotation of the earth about its own axis and

the orbit of the earth about the sun introduce apparent motions to the observer on the earth of the fixed star field surrounding the earth and these apparent motions are corrected for in any observations made by an observer on the surface of the earth or flying in a conventional aircraft near the surface of the earth. The basis of celestial navigation is that the angular measurement of the positions of known fixed stars and the determination of the angles of a given set of star configurations for a given point on the surface of the earth uniquely determines that point. Due to the vast astronomical distances at which the star fields are located however, the angular locations of the stars will not be changed by a mere translation in space within the vicinity of the earth of the order of solar system distances. Accordingly, a coordinate system for the earth and the celestial globe surrounding the earth will apply equally as well to an orbiting space vehicle if the angular orientation between the celestial sphere and a coordinate grid is the same as that employed on the earth's surface. As a final step, of course, the earth's longitude coordinate will require to be corrected for time of day with reference to the Greenwich meridian as is well known by adding Greenwich hour angle of the first point of Aries (♈) to the sidereal hour angle. The present invention utilizes these facts in the manner hereinafter described.

FIG. 2 shows an elementary form of navigation device usable in accordance with the invention which comprises a transparent hemisphere **14** which may be of suitable material such as methyl methacrylate, for example. On the spherical surface of the hemisphere **14** are inscribed declination circles **15** and sidereal hour angle arcs **16**. Also inscribed on the surface of the hemisphere **14** are members of the celestial star field for a particular hemisphere properly positioned with reference to the celestial sphere and the grid formed by the circular lines **15, 16**. A suitable number of stars are inscribed on the hemisphere **14** and identified by name, for example, to provide a sufficient number for navigational purposes as may be required. The inscription of the grid **15, 16** and the star field with the identifying names of the stars may be in the form of an engraving such that in combination with the material forming the hemisphere **14** the grid and star field may be illuminated by well known edge lighting techniques as will hereinafter be set forth. As used for space navigation, the apparatus of FIG. 2 will appear to the observer's eye located at the center of the hemisphere much as it appears drawn in FIG. 2 and in use the apparatus will be directed in the direction of the earth which will appear as a disc **12** in the field of vision.

FIGS. 3 and 4 show a suitable arrangement for utilizing the navigational device of FIG. 2. For this purpose a plurality of support arms **17** are arranged with spring clips **18** at the extremities of the arms **17** and supporting a face support member **19** at a central location relative to the hemisphere **14**. The face support member **19** may be any suitable device for accurately positioning an observer's eye at the center of the hemisphere **14**. Many well known optical devices use fine cross hairs as a positioning index. In the detail of FIG. 5 the clips **18** are shown engaging the wall of the hemisphere **14** and are provided with a lamp housing **21** containing a suitable source of light **22** for edge lighting the hemispherical shell **14**. Switch means **23** may be provided for selectively illuminating the sphere **14**. Any suitable source of energy for the illumination means **22** may be employed such as a self contained battery as indicated at **24** in FIG. 3.

The details of supporting the face rest **19** on the arms **17** are shown in FIG. 6 wherein the support rods **17** terminate in a circular race **25** supporting a plurality of balls **26** spaced around the race **25**. By suitable means the assembly **25, 26** may be magnetized in order to attract the circular frame **27** which supports the face pad **19**. The member **27** is shaped with a curved inner surface **28** formed to fit the balls **26** and provide a ball bearing type relative movement between the members **27** and **25**. With this arrangement the entire apparatus may be freely rotated about the eye of the observer when he positions his face on the pad **19** to center one eye at the center of the hemisphere **14**. For convenience the assembly **27, 19** may be readily removed by applying sufficient force to overcome the magnetic attraction of the magnetic means provided in the race **25**.

An arrangement for employing the method of the invention to obtain an indication relative to the actual earth latitude and longitude is shown in FIG. 7. For this purpose the star globe **14** may be provided with a central bushing **31** and a grooved raceway **32** near the equatorial plane. A second concentric hemisphere **33** of slightly smaller radius than the hemisphere **14** is arranged to be placed within the hemisphere **14**. The hemisphere **33** has a grooved raceway **34** to cooperate with a plurality of balls **35** spaced around the equatorial plane of the spheres for rotatably supporting the spheres **14** and **33** relative to each other. As shown in FIG. 8, the sphere **33** has a keyed aperture bushing **36** at the pole position opposite bushing **31**. The hemisphere **33** may be of suitable transparent material and has a representation of the earth's land masses thereon and generally resembles the hemisphere from a conventional globe of the world. The sphere **33** may also be edge lighted from the same light source as employed to edge light the sphere **14**. With this arrangement, the correct relative rotative position of the spheres **14** and **33** will convert the sidereal hour angle arcs **16** and declination circles **15** into longitude and latitude coordinates respectively on the representation of the earth on globe **33**. The correct relative position of hemispheres **14** and **33** is maintained by a sidereal hour clock **37** which may be connected by a flexible shaft drive to the bushing **31** and keyed aperture bushing **36** in the sphere **33**. The flexible shaft **38** terminates in a keyed bushing **39** which fits the bushing **31** in the sphere **14** and the drive shaft **40** within the flexible cable **38** is keyed to fit the keyed aperture **36** in the sphere **33**. Accordingly, the keeping of correct time on a sidereal clock **37** will automatically result in the spheres **14** and **33** being correctly relatively positioned by merely inserting the flexible cable **38** into the bushing **31** with the cable bushing **39** in registry with the key means therein and rotating the inner sphere **33** until the keyed aperture **36** fits the keyed drive shaft **40**. The assembled relation may be maintained by detents or other suitable means.

Referring now to FIG. 9 a modification of the device usable in practicing the invention is shown. For relatively low altitudes of the space vehicle the earth disc will occupy a wide angle of the field of vision and accordingly it may be possible that the stars observed beyond the horizon of the earth may be at extreme angles approaching the equatorial plane of the star globe **14**. For these conditions it may be desirable to modify the navigational device by providing in the race **25** a planetarium type sphere **41** which may be magnetically retained in a universally rotatable position against the balls **26**. The sphere **41** may be a hollow shell with a suitable sidereal hour angle and declination grid formed therein in the form of perforated lines the perforations not being continuous in order to maintain the physical structure of the sphere **41** intact. Also accurately located on the sphere **41** are perforations and identifying marks corresponding to the navigational stars. Within the sphere **41** a suitable light source is provided with means for turning the light source on and off and, if desired, a self contained power supply such as a battery. With this arrangement and the light source energized the sphere **41** will project on the inner surface of the hemisphere **14** a coordinate grid and points of light representing the various stars used for navigation and an identification thereof such as the names of the stars. By rotating the sphere **41** in its mount until the points of light corresponding to particular stars are in registry with the actual star image on the surface of the sphere **14**, a correspondence will be obtained between the coordinate grids projected on the inner surface of the sphere **14** and the coordinates on the earth as viewed

through the sphere **14**. For the purposes of obtaining images of the stars in the star field on the hemisphere **14** a suitable optical device such as an annular lens **42** may be provided to focus images of the stars on the hemisphere **14**. Since this modification of the apparatus used may find primary utility for stars located at wide viewing angles, the lens **42** is indicated as being an annular lens suitable for producing star images within 20° approximately of the equatorial plane. Under these conditions it will be advantageous to employ the circles of declination **15** to center the earth image therein in order that the pole **43** of the hemisphere **14** may be used as a ground point index as will be hereinafter explained. FIG. 10 shows an alternate sphere having suitable hour angle and declination grids imprinted thereon which may be used in place of sphere **41** of FIG. 9.

When in space, an observer requires a three dimensional position. To determine his altitude vertically above his geographical position, the space observer measures as indicated in FIG. 11 with a marine sextant, an iris shutter, a peripheral scanning device, or by any of several available means, the angle subtended by the earth's disc. With this observed angle as an argument, together with the earth's diameter, the required third dimension, altitude, in terms of nautical miles vertically above the observer's geographical position may be computed by standard trigonometrical methods. Additionally, a standard table may be prepared which indicates altitudes for various angles subtended by the earth's disc. Thus, the observer would have available a quick reference upon determination of the subtended angle.

The method of the present invention will be described with reference to the apparatus disclosed. Referring to FIGS. 3 and 4 for the use of the hemisphere described in detail in FIG. 2, the observer will place the face against the rest assembly **19** in the rotatable mount **25** and thus place his eye in the center of the hemisphere **14**. Referring now to FIG. 2, the view to the observer will generally be that of looking through a hemisphere with the appropriate grid markings thereon and the device will be directed so the observer may look in the general direction of the earth **12**. By selecting suitable navigation stars in the hemisphere in which he is looking the observer can rotate the hemisphere **14** and change the polar direction of the hemisphere **14** until two or more observed stars coincide with the engraved representations of the same stars on the surface of the hemisphere **14**. For this purpose the stars may be illuminated by energizing the lamp **22** with the switch **23** to edge light the engraved indices on the plastic hemisphere **14**. For these conditions, the grids **15** and **16** and the stars marked thereon will be illuminated and clearly discernable to the operator. Once the actual stars have been observed as superimposed on the representations thereof engraved on the hemisphere **14**, the observer's position relative to the earth can be read directly from the scales associated with the grid **15, 16**. The exact position will correspond with the center **44** of the earth disc **12**. It should be noted that the reading of the scales associated with the grid **15, 16** will correspond to those of the opposite hemisphere to that over which the observer is located. In other words the celestial hemisphere and the stars on which the observer is taking a fix correspond to that hemisphere which is on the far side of the earth with respect to the observer. However, to know his position above the earth as diametrically opposite that position located in the celestial hemisphere on the far side of the earth. Accordingly, the grid **15** and **16** may be designated with index numbers in which observed declination is opposite as to North and South designation that of the observed stars and sidereal hour angle is the sum of the sidereal hour angle plus 180° of the observed stars for the northern hemisphere of celestial bodies depicted in FIG. 2. The actual position of the vehicle which would be occupied when using the hemisphere would

correspond to the grid markings in the southern hemisphere for declination and with the sidereal hour angles marked as shown in parentheses. Obviously, this correction could be interpreted by the observer with the scales marked in the usual manner for the hemisphere being observed.

The operation of the device of FIG. 7 corresponds generally with that just described with reference to FIG. 2 except that the sidereal clock **37** has been suitably engaged to drive the earth globe **33** relative to the hemisphere **14**. Upon taking an observation with the apparatus of FIG. 7 the position of the earth disc **12** and in particular the center thereof **44** will be located at the ground point of the space vehicle on the earth globe **33**. Since the earth globe **33** is provided with representations of the land masses and bodies of water of the earth in a normal manner, the space navigator can recognize his location on the surface of the earth by observing the center **44** of the earth disc **12** relative to the actual representation of the earth on the globe **33** and the latitude and longitude coordinates.

In using the apparatus of FIG. 9 the preferred arrangement would be to center the earth disc **12** in the declination circles **15** in order that the pole **43** would accurately represent the center **44** of the disc **12**. With this arrangement the projected points of light representing stars and the grid representing declination and sidereal hour angle are variably positioned on the inside surfaces of the hemisphere **14** upon projection from the sphere **41**. The lens **42** focuses images of the real stars on the surface of the hemisphere **14** and by rotating the globe **41** coincidence between known navigational star images and the projected points of light representing the stars is obtained. For this condition, the grid then projected on the earth disc observed as centered in the meridian lines **15** will give the accurate location of the space vehicle **11** with the ground point or geographical location of the vehicle indicated by the pole **43** on the projected grid coordinates.

To summarize, a method utilizing simple, manually operated, lightweight means has been described for fixing the position of a space vehicle directly, continuously, and in units of latitude, longitude and altitude in nautical miles, with which the space navigator will be familiar.

When the observer is so far from earth that its rotation might be ignored, the observed sidereal hour angle and latitude of his geographical position would suffice. The supplemental globe **33** driven by a sidereal clock **37** merely adds Greenwich hour angle of Aries continuously and automatically. Alternatively, the observer could find this value from the air almanac for any desired instant and add this to his observed sidereal hour angle to determine his longitude.

The method described might be compared with the technique of piloting in marine navigation where the bearing of and distance from a lighthouse are well known. In the present method a way is provided for fixing the geographical position of the latitude and longitude of the observer, which is the point where the line from the observer to the center of the earth pierces the earth's surface. A simple way has been disclosed for determining the altitude of the observer vertically above his geographical position, and this, in familiar terms.

While particular devices usable in practicing the invention and methods of operating same have been disclosed it will now be apparent to those skilled in the art that various modifications thereof may be made. The invention is not to be limited to utilization of the specific devices shown by way of illustration but only by the appended claims.

I claim:

1. The method of finding the ground point of a space vehicle comprising the steps of observing the earth and the star field beyond the earth in the direction of observation, orienting a sidereal hour angle and declination

angle grid with at least two known stars in said star field and reading the location of ground point on said grid at the center of the earth as observed from said vehicle.

2. The method of finding the latitude and longitude of the ground point of a space vehicle comprising the steps of observing the earth and the star field beyond the earth in the direction of observation, establishing reference coordinates oriented with at least two known stars in said star field, rotatably displacing a latitude and longitude grid from said reference coordinates by the Greenwich hour angle of the first point of Aries, and reading the latitude and longitude of ground point on said grid at the center of the earth disc as observed from said vehicle.

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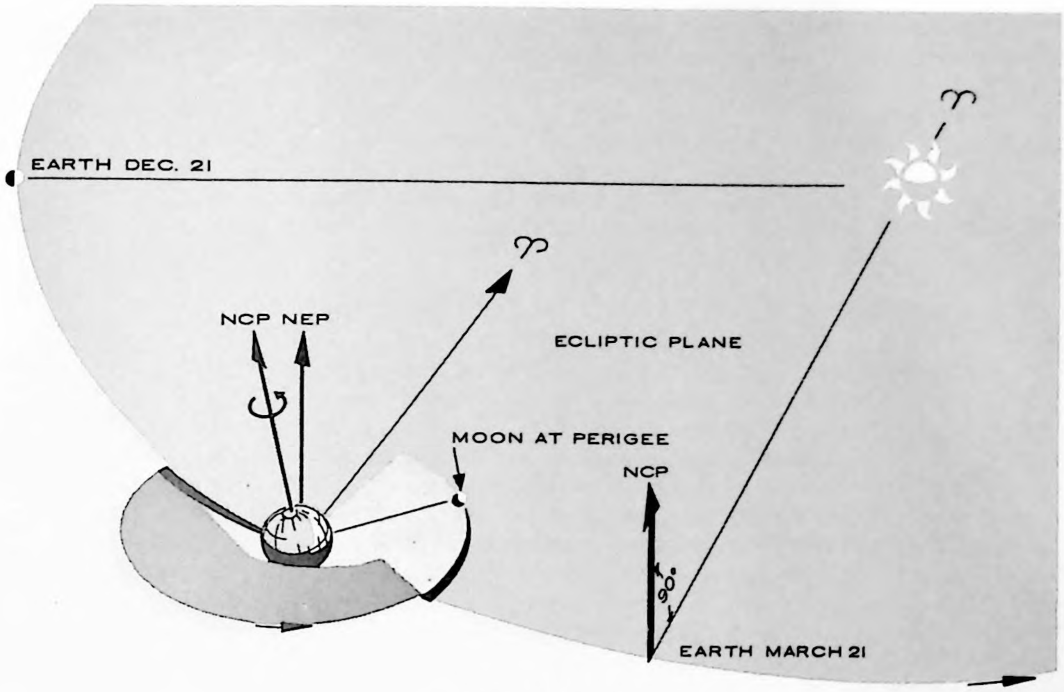

APPENDIX B

Introduction to the MOTIONS OF THE MOON*

Joseph P. Duda

Summary — As space travel advances from theory to practice, more and more engineers must concern themselves with the complex motions of the moon and earth in relation to the sun. The language used to describe these motions, heretofore the property of the astronomer and astrophysicist, is offered in this paper to the engineer.

INTRODUCTION

Since the moon is by far the closest celestial body to us, it is the principal celestial goal for space vehicles. Trajectories for the purpose of hitting the moon, landing on it, or orbiting a vehicle around it are described in the literature of space navigation, and its terms are familiar to engineers directly concerned with the subject. Engineers in other fields, however, may not be as conversant with astronautical nomenclature and with the involved motions of the moon. It is the purpose of this paper to bridge the gap between the reader's present knowledge and the fund of information assumed for the reader of the more specialized literature.

In Fig. 1, at the top of this page, a portion of the earth's orbit is seen from the north side (the side toward Polaris). Motions that appear counterclockwise in this view are defined as eastward motions. The earth orbits in the ecliptic plane, while the moon orbits in a plane tilted five degrees away from the ecliptic. These planes intersect in the line of nodes.

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The motions of the moon may be divided, for the sake of convenience, into four general categories: the motion of the earth-moon system around the sun, the changing orientation of the moon's orbital plane, the orbiting of the moon within this plane, and the motions of the moon about its own axis. The rotation of the earth about its own axis also contributes to the moon's apparent behavior as seen from the earth. Each of these phenomena and the terms and geometrical coordinates that are conventionally used will be discussed in this paper.

THE EARTH AND MOON IN THE SOLAR SYSTEM

The earth revolves once a year around the sun in an elliptical orbit (Fig. 1). The radius of this orbit varies from about 91.5 to 94.5 million miles. The plane in which the earth's orbit lies is, for practical purposes, fixed in space, and is called the ecliptic plane. The side of the ecliptic plane toward the earth's north pole may be considered to be the north side. Looking at this north side, both the revolution of the earth around the sun and its rotation around its own axis are counterclockwise; these or any other rotations or revolutions in the same direction are defined as eastward motions. Fortunately, all the motions to be described take place about axes that are within 25 degrees of being perpendicular to the ecliptic plane; therefore there is no difficulty in applying these definitions to all of them.

The moon revolves about the earth in an elliptical orbit having a mean radius of 240,000 miles, or about 1/400 the size of the earth's orbit. This motion is also eastward, or counterclockwise as seen from the north.

THE CELESTIAL SPHERE

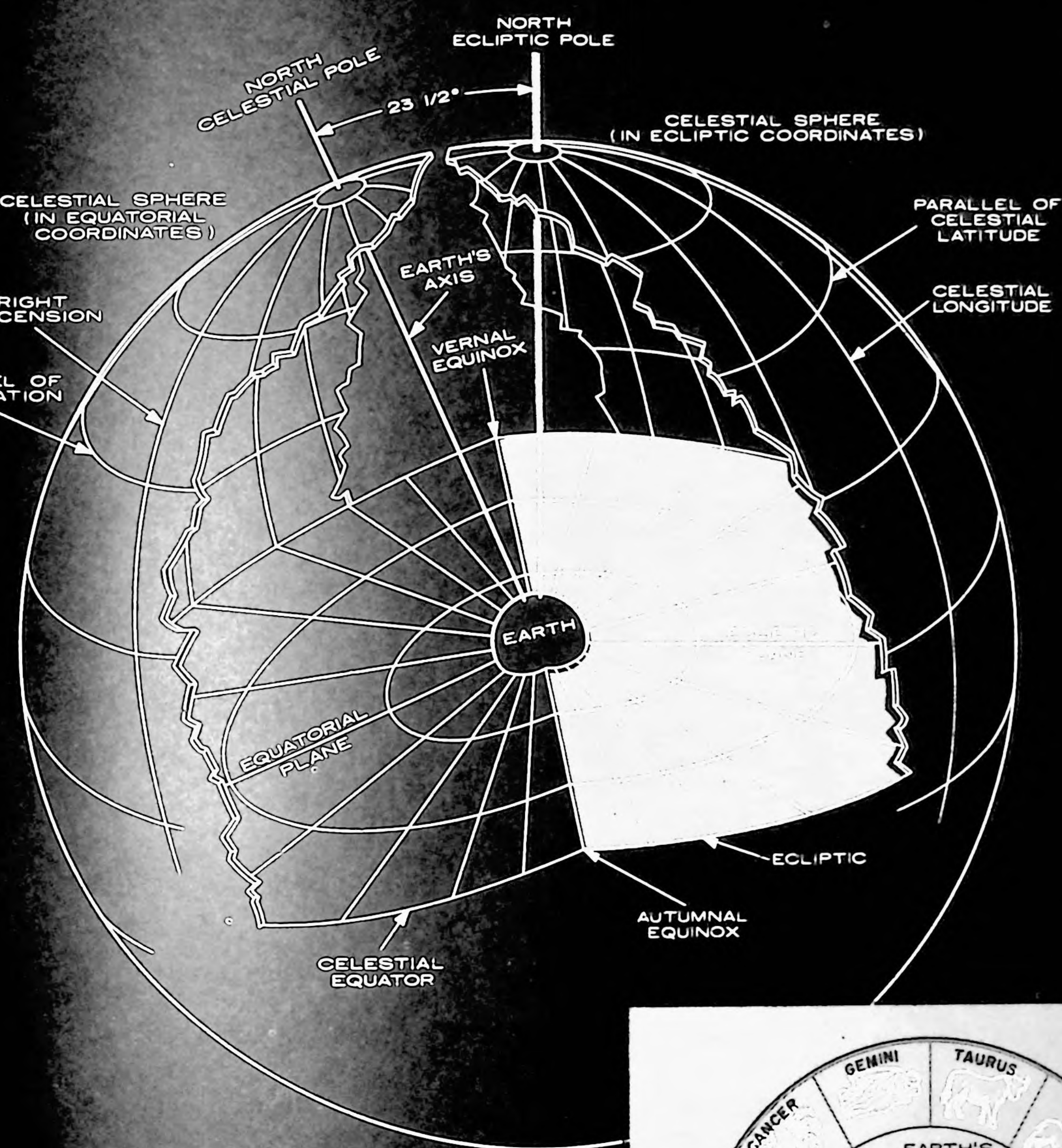
For the purpose of identifying directions in space, it is customary to imagine that all of the distant stars lie on a vast sphere, called the celestial sphere, which has its center at the earth's center as depicted in Fig. 2. Although, the stars are moving very rapidly, they are so far away that their relative movements escape detection except by measurements of extreme precision.

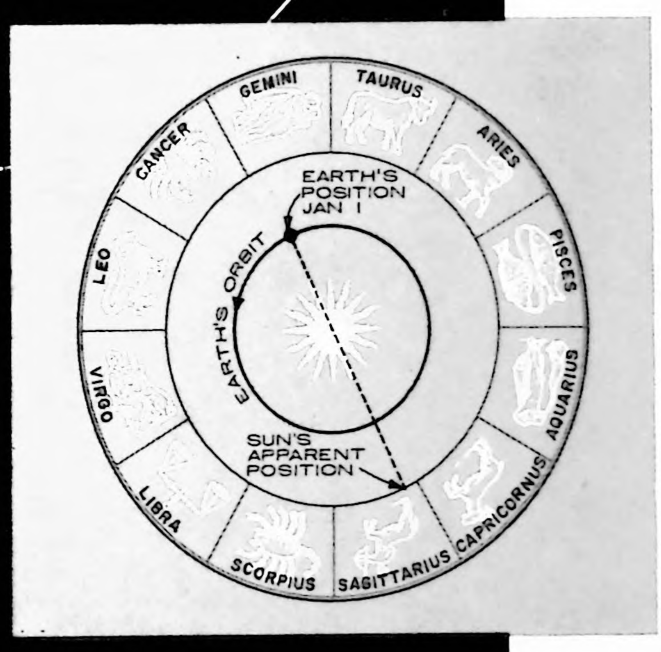
On the celestial sphere two great circles, known as the ecliptic and the celestial equator, are used as reference circles for the ecliptic and equatorial coordinate systems respectively. The ecliptic is formed by the intersection of the earth's orbital plane with the celestial sphere. It is the path of the sun's apparent annual motion as seen from the earth (Fig. 3). The celestial equator is formed by extending the plane of the earth's equator (the equatorial plane) until it intersects the celestial sphere.

The ecliptic and celestial equator intersect at two points called the equinoxes (Fig. 2). The vernal equinox, which is used as a reference point, is that point of intersection at which the sun, in its apparent motion along the ecliptic, crosses the celestial equator moving from south to north. This happens on about March 21 of each year. The sun appears at the other point of intersection, called the autumnal equinox, on about September 21 each year, this time crossing the equator from north to south. On these two dates the earth's axis of rotation is perpendicular to the radius vector from the sun to the earth, as seen from Fig. 1.

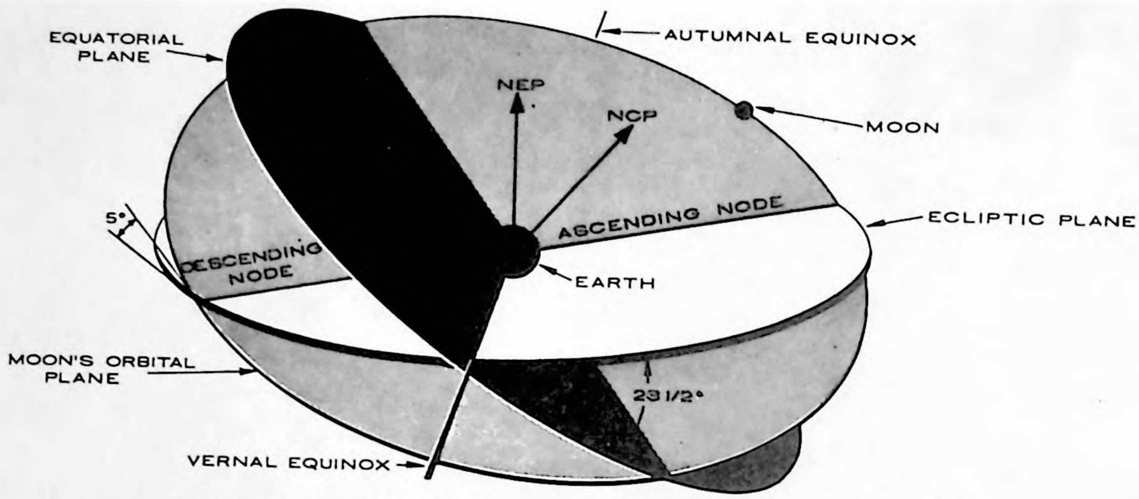
In the ecliptic coordinate system, points on the celestial sphere are identified by celestial latitude, measured northward or southward from the ecliptic and by celestial longitude, measured eastward and westward along the ecliptic from the vernal equinox (Fig. 2). A line through the center of the earth and perpendicular to the ecliptic plane intersects the celestial sphere at two points called the north and south ecliptic poles (NEP and SEP).

In the equatorial coordinate system a point on the celestial sphere is located by declination and by right ascension. Declination, measured in degrees northward or southward from the celestial equator, corresponds (except for the minor effects of gravitational deviation and of the earth's





The earth-centered celestial sphere depicted in Fig. 2, above, is used to define the positions of bodies in space. The coordinate systems are based on the ecliptic and equatorial planes. Measurements in right ascension and declination are used in earth-based observations; those made in celestial longitude and latitude are used in calculations where the orientation of the earth's rotation is unimportant. Fig. 3, right, shows how constellations are used to define the position of the ecliptic plane.



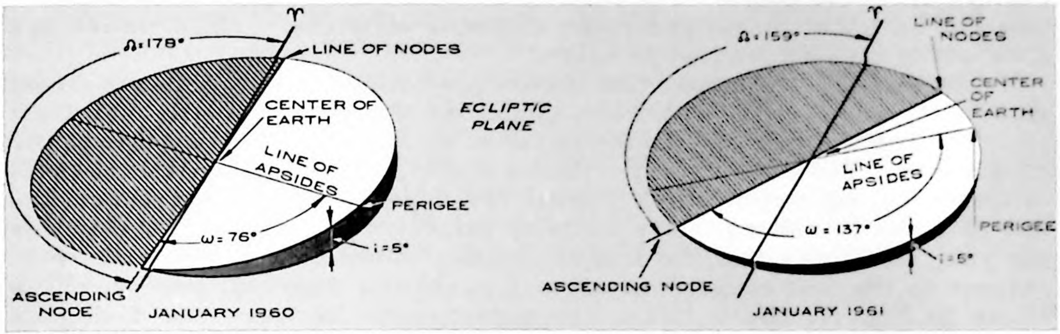
The typical orientation of the moon's orbital plane, the ecliptic plane (the earth's orbital plane), and the equatorial plane is shown in Fig. 4, above. Angles between the planes are constant. The special case, in which the planes meet in a common intersection, occurs only once in 18-1/2 years, when the regression of the line of nodes makes it coincide with the line of equinoxes.

slightly nonspherical shape) to the parallel of latitude on the surface of the earth along which the given celestial point passes directly overhead. Right ascension is measured eastward along the celestial equator from the vernal equinox. It is usually expressed in hours, minutes, and seconds, from 0 and 24 hours, where 1 hour equals 15 degrees of arc. The earth's axis of rotation intersects the celestial sphere at the north and south celestial poles (NCP and SCP). The north celestial pole remains at one spot on the celestial sphere, close to the star Polaris and 23-1/2 degrees away from the north ecliptic pole, at all times of the year. It actually moves westward through the sky, circling around the north ecliptic pole once every 26,000 years. This effect and the consequent movement of the equinoxes, are very slow compared with the other motions described in this paper.

THE PLANE OF THE MOON'S ORBIT

The plane in which the moon's elliptical orbit lies is inclined about five degrees from the ecliptic plane, as shown in Fig. 4. These two planes intersect in a line called the line of nodes. Unlike the ecliptic plane, however, the moon's orbital plane does not maintain a fixed position in space, but precesses about an axis perpendicular to the ecliptic plane. For this reason the line of nodes regresses (moves westward), completing a circle once every 18-1/2 years (Fig. 5). Therefore the moon's orbital plane, the ecliptic plane, and the equatorial plane are usually skew to one another, as in the typical case shown in Fig. 4. The only time that all three intersect in a common line is when the line of nodes coincides with the line of equinoxes, once every 18-1/2 years.

The ascending node is the point on the line of nodes where the moon crosses the ecliptic plane moving from south to north. The descending node is the point, at the opposite end of the line, where the moon again crosses the ecliptic but is moving from north to south. To describe fully the orientation of the moon's orbital plane at any instant, it is necessary to know two angles, as illustrated in Fig. 5. These are the inclination (i) of the orbital plane to the ecliptic, which remains constant at about five degrees, and the celestial longitude of the line of nodes, measured eastward along the ecliptic from the vernal equinox to the ascending node.

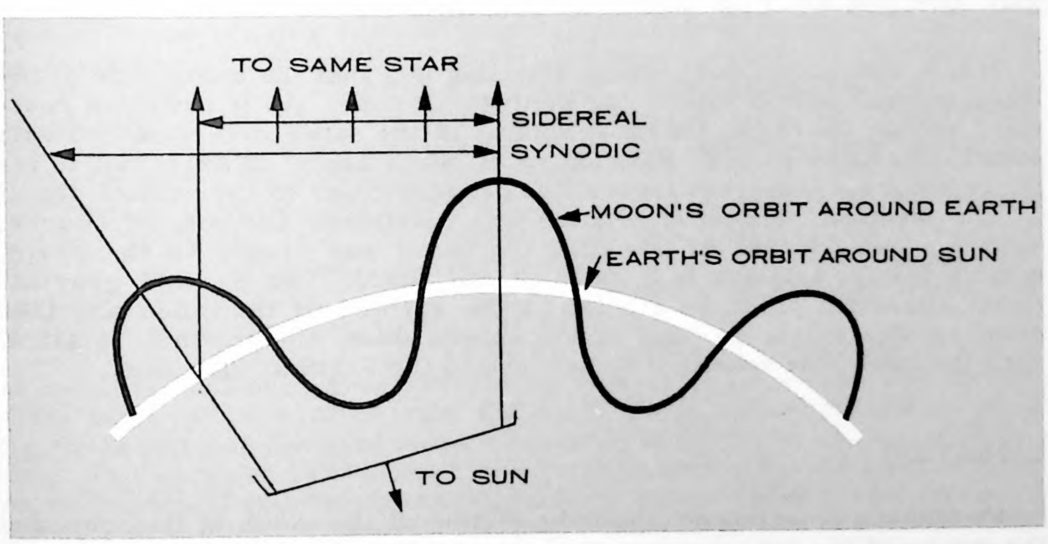


The yearly change of position of the line of apsides and the line of nodes, caused by their rotation in opposite directions, is shown in Fig. 5, above. Measuring the angle between the line of apsides and the line of nodes, and the angle between the latter and the vernal equinox, gives the instantaneous position of the orbit of the moon.

THE MOTION OF THE MOON WITHIN ITS ORBITAL PLANE

On some evening, note the position of the moon against the background of the stars. At the same time next evening, it will be about 13 degrees farther east. The moon moves eastward in an elliptical orbit, completing one revolution with respect to the "fixed" stars in about 27-1/3 days. This time is called its sidereal period. As the moon revolves around the earth, it changes its apparent position with respect to the sun, causing the changing illumination that we call the phases of the moon. The time from full moon to full moon is about 29-1/2 days. This time is called its synodic period, and is longer than the sidereal period (Fig. 6) because the moon accompanies the earth in its annual revolution around the sun. A third period, the draconitic period of about 27-1/5 days, is the interval between the moon's crossings of the ecliptic plane at the ascending node (Fig. 4).

Since the mass of the earth is only about 81.3 times that of the moon, the center of the earth cannot be taken as the center of gravity of the earth-moon system. The center of mass of the system lies inside the earth, about 1,000 miles below the surface and on the line between the



As shown in Fig. 6, above, the moon must orbit more than 360 degrees between successive full moons, since the moon follows the earth and the relative positions of earth and sun change. Thus the synodic period exceeds the sidereal.

centers of the two bodies. The earth and the moon both revolve about this common center of mass, generating confocal elliptical orbits which are different in size but similar in shape.

It has already been stated that the moon's orbital plane precesses with an 18-1/2-year period. Within this plane, the major axis of the elliptical orbit (called the line of apsides) rotates in an eastward direction, or progresses. The line of apsides makes a complete rotation with respect to the vernal equinox in about 9 years (40 degrees per year). Since the line of nodes is moving in the opposite direction at roughly 20 degrees per year, it follows that the line of apsides completes one rotation with respect to the line of nodes in about 6 years (60 degrees per year), as shown in Fig. 5. These values are approximate because the 5-degree inclination of the orbital plane to the ecliptic plane has been neglected.

The end of the major axis at which the moon comes closest to the earth is called the perigee, while the opposite end, where the moon is farthest away, is called the apogee. The instantaneous position of the moon's orbit is defined by the longitude of perigee (Γ'), which is expressed as the sum of two angles: the longitude of the line of nodes, measured in the ecliptic plane as already described, and a second angle, measured in the orbital plane from the ascending node eastward to the lunar perigee (Fig. 5). The instantaneous longitude of the moon itself is expressed in the same way, the second angle being measured from the ascending node to the moon.

THE MOON'S DECLINATION

From the combination of skew planes and elliptical orbits previously described, it is obvious that prediction of the moon's position in equatorial coordinates for a particular time requires tedious computations. It is not necessary to make these computations since the results are available in almanacs. However, it may be noted that the moon reaches a maximum northward declination once during each draconitic period, when it crosses a line perpendicular to the line of nodes. This maximum northward declination can never be higher than 23-1/2 + 5, or 28-1/2, degrees, and this value can occur only once in 18-1/2 years, when the ascending node coincides with the vernal equinox (Fig. 4). Similarly, the southward declination of the moon reaches a maximum of 28-1/2 degrees when the descending node coincides with the vernal equinox.

THE MOTION OF THE MOON ON ITS AXIS

It is a well-known, but perhaps startling fact that the same side of the moon remains turned toward the earth at all time. As it revolves eastward around the earth, the moon rotates in the same direction and with exactly the same period. Rotation takes place about an axis that is inclined about 6-1/2 degrees from a line perpendicular to the orbital plane.

The identical periods of rotation and revolution are not, of course, simply a coincidence. At one time the moon was closer to the earth, rotated faster, and was in a more plastic state. The earth's gravitational attraction produced a bulge at the surface of the moon and then acted on this bulge, as tidal action slowed down the moon's rotation, until the same side remained turned toward the earth at all times.

LIBRATION

To obtain a lunar impact, both the motion of the moon in its orbit and the motion of the orbital plane must be considered. However, to attain an accurate soft lunar landing at a predetermined location, the lunar librations (oscillations of the moon on its axis) must also be considered. The two librations affecting lunar vehicles are libration in latitude and libration in longitude.

Libration in latitude results from the 6-1/2-degree inclination of the moon's equator to its orbital plane. As the moon revolves around the earth the axis of the moon remains parallel to itself. Consequently a point on the surface of the moon oscillates toward and away from the orbital plane, as shown in Fig. 7. The total angular oscillation in latitude is about 13 degrees, or twice the inclination of the moon's equator to the orbital plane. To an observer on the northern hemisphere of the moon, the earth would appear higher in the sky when the moon is in the position shown at the left in Fig. 7, and lower when it is in the position shown at the right. This is similar to the relationship between earth and sun that produces our seasons.

In Fig. 7, below, points A and B are fixed on the moon's surface. Because the moon's axis of rotation is not perpendicular to its orbital plane, points A and B oscillate toward and away from the plane once per orbit.

Libration in longitude arises from the eccentricity of the moon's orbit. The moon rotates once in each revolution. This means that the rotational period of a radius of the moon equals the rotational period of the line connecting the centers of the earth and moon. Although the periods of these two motions are the same, they do not always keep pace with one another. Since the moon's orbit is elliptical, its angular velocity of revolution varies as it revolves around the earth. The rotation of the moon on its axis, however, is more nearly uniform. As the angular velocity of revolution drops below the rotation rate, we begin to see more of the western limb (edge): that is, a point on the surface of the moon appears to be moving toward the east. During the period of revolution, the moon appears to oscillate about its axis of rotation, as illustrated in Fig. 8. This oscillation amounts to about 7-3/4 degrees each way. Although both of the oscillations described are not actual oscillatory motions of the moon but simply arise from the geometry, they appear as oscillations to a vehicle on a lunar trip.

Physical libration results from an actual irregularity in the moon's rotation about its axis. The moon is not spherical, but has a bulge of about 5/8 mile directed toward the earth. The earth's gravitational attraction on this bulge causes a slight oscillation of the moon equivalent to about a mile at the surface of the moon.

Before concluding, it may be well to hold a brief qualitative review of the motions that have been described. Observing the large-scale motion of the earth-moon system from the north side of the ecliptic plane, as in Fig. 9, we see that the predominant tendency is eastward or counterclockwise. The earth and the moon rotate (turn on their own axes) and revolve (move in orbit) counterclockwise, and the line of apsides progresses in this direction also. However, the regression of the line of nodes and the precession of the earth's axis both occur in the clockwise (westerly) direction.

The moon, in the course of its annual journey around the sun, makes about 13 sidereal revolutions around the earth, and its orbit always remains concave toward the earth. Surprisingly, it also remains concave

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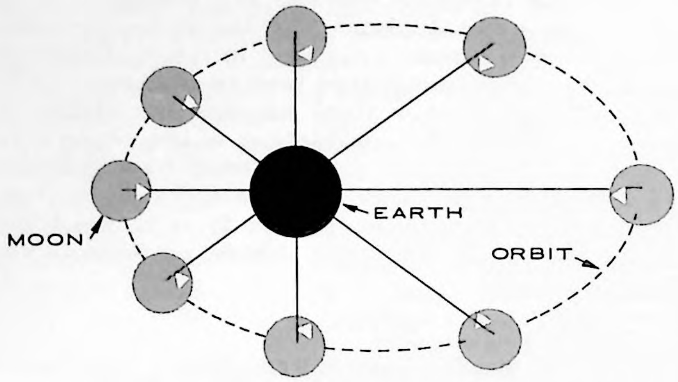
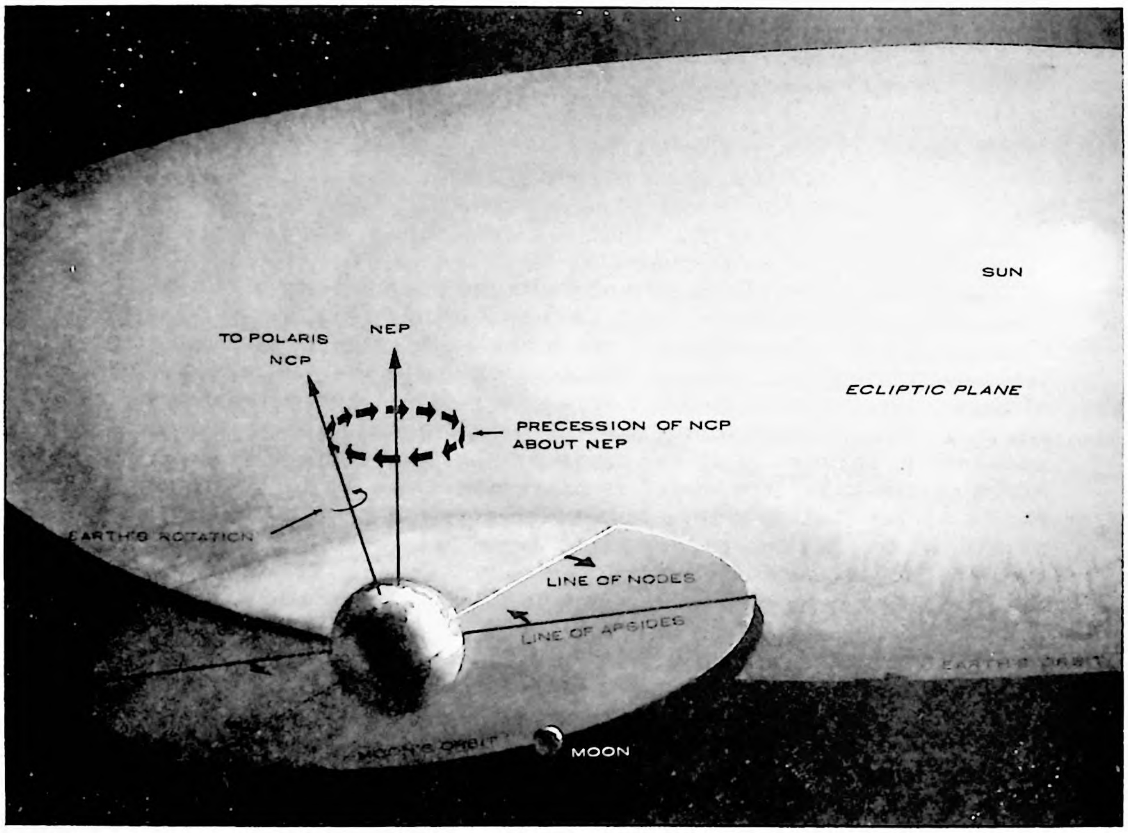


Eccentricity of the moon's orbit causes libration in longitude, shown in Fig. 8, above. Thus a point fixed on the moon oscillates one cycle per orbit about the axis of rotation. Motions discussed in the paper are shown in Fig. 9, below; all are eastward except for the precession of the earth's axis of rotation and the regression of the line of nodes.



toward the sun, because of the moon's smaller orbit and particular orbital period (Fig. 10). This fact is not apparent in Fig. 6, where several revolutions of the moon have been compressed into a small part of the earth's orbit.

CONCLUSION

This paper has attempted to introduce the reader to the general picture of the motions of the moon and the earth. A particular effort has been made to clarify the relationship between the actual motions of these bodies

The curious fact that the moon's orbit is always concave toward the sun is demonstrated in Fig. 10, above. A line drawn perpendicular to a radius from the sun at any point in the orbit of the moon around the earth will fall outside the path traced by the moon as it travels with the earth about the sun.

through space and their appearance to an observer on the earth. The ecliptic and equatorial coordinate systems, the longitude of perigee, and the other terms that have been defined here, may soon become as well-known as are the terms used to give directions and positions on the earth.

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The editor of Sperry Engineering Review has given permission to use material from J. P. Duda's Article which appeared in the December 1960 issue. Since this article is pointed to the engineer, is beautifully illustrated in color, and is a rounded story in itself, it is included here as Appendix B, although it parallels portions of the text.

APPENDIX C

Letters which brought about the Pilot Class in Space Navigation

DEPARTMENT OF THE NAVY
Office of the Chief of Naval Operations
Washington 25, D. C.

Op-762/orw
Ser 24P76
10 Dec 1959

From: Chief of Naval Operations
To: Chief of Naval Personnel

Subj: Exploratory Development of Space Piloting Training Techniques

Ref: (a) Operational Requirement No. AD-01502 "Sea Based Manned Maneuverable Intercepter"

1. With the initiation of projects such as MERCURY and DYNA SOAR it is apparent that manned space flight will be attempted in vehicles having increasing capability of maneuver and range from the earth's surface. Despite distances involved in such flights, the necessary base for navigation or space piloting is the earth or the earth-moon system. Hence, the ability to navigate in the near earth region with respect to earth coordinates is of primary importance.

2. Reference (a) states the long range Navy requirement for manned spacecraft. More recent is the Department of Defense assignment of the satellite navigation development responsibility to the Navy. While this effort is currently directed to systems for use by surface ships, the logical follow-on includes aircraft and spacecraft navigation.

3. Projected plans for near-earth navigation involve the use of computers, surface detection systems, and electromagnetic communication and data links. These are very complex systems which involve either two or three coordinate transformations to obtain knowledge of position, velocity and vehicle attitude. They are not 100% reliable.

4. Preliminary discussion with Captain P. V. H. Weems, USN (Ret), at Annapolis, have indicated that very simple techniques may be employed from the space vehicle by a human pilot to obtain information on position and vehicle attitude. Point by point extrapolation, perhaps with simplified analogue aids, would permit some knowledge of velocity vector relative to earth.

5. It is requested that the Bureau of Naval Personnel consult with Captain Weems and such others as may be appropriate to ascertain the requirement for an exploratory development of space piloting training techniques. Inasmuch as the results of such investigation may have considerable effect on the direction and instrumentation costs of future space programs, it is requested that any resulting program be informally reviewed by the deputy Chief of Naval Operations (Development).

/s/ Chas B Martell

CHAS B MARTELL
By direction

18616

Pers-C114-blb
Ser: C114/99.9
19 February 1960

From: Chief of Naval Personnel
To: Chief of Naval Operations (OP-762)

Subj: Exploratory Development of Space Piloting Training Techniques; requirement for

Ref: (a) CNO ltr to CNP OP-762/orw Ser 24P76 dtd 10 Dec 1959
(b) OPNAV INSTRUCTION 5500.33 (Subj: Department of the Navy Long Range Scientific and Technical Planning Program)

Encl: (1) Captain Weems' Space Piloting (navigation) Course Outline
(2) List of persons consulted on the subject
(3) Brief Space Navigation Concept as expressed by Captain Weems

1. Reference (a) requested that the Chief of Naval Personnel consult with Captain P. V. H. Weems, USN (Ret) of Annapolis and such others as might be appropriate to ascertain the requirement for an exploratory development of space piloting training techniques. Subsequently, a representative of the Chief of Naval Personnel met with Captain Weems to discuss his approach to space piloting techniques.

2. The Weems' approach to space piloting (navigation) emphasizes simplicity. It assumes that a manned space vehicle can be placed into orbit around the Earth and that the man in the vehicle will have the capability of influencing the attitude of that vehicle to the extent that observations may be made. In the situation thus described, the man determines his position in space by the observed position of the Earth's center relative to the positions of known stars. He navigates in the Earth-Moon system by a simple method of fixes which could be termed piloting, though the word is not to be misconstrued as to suggest driving or propelling the space vehicle. This method of space navigation, the determination of course by continually recomputing the vehicle's instantaneous position, would require much simpler equipment than one of determining a course and then expending all effort to make good a track over this course. Captain Weems suggests that we learn the simple method first and proceed to more advanced techniques as we gain greater experience and capability. He has prepared a course outline (Enclosure (1)) and is also writing a book on the subject.

3. As suggested in reference (a) other personnel have also been consulted regarding space piloting training techniques. A list of these persons is included as enclosure (2). In all discussions, the Weems' approach to the subject was used. The consensus was that some preliminary or orientational space piloting training is worthy of further pursuit, even though navigational techniques for this area may still be in their infancy. The problems that seem formidable today may appear simple in retrospect as man meets the challenge of space creatively.

4. In view of the foregoing, it is recommended that:

a. The Chief of Naval Operations initiate a project for development of an orientational course in the theory of space piloting, following the brief outline provided in enclosure (1) and developing the concept expressed in enclosure (3).

b. The Chief of Naval Personnel provide support in the course development in regard to training aids to the extent that these are presently available and will not require extensive new development.

c. The course be reviewed, when completed, by the DCNO (Air), DCNO (Development) and the Chief of Naval Personnel with a view to utilizing it experimentally at a place to be determined, possibly Pensacola, Florida.

d. If the course proves satisfactory, it be given to selected officers based on Navy needs.

5. During informal discussions in OP-762, Captain Dare suggested that the Chief of Naval Operations (OP-762) invite Captain Weems to execute enclosures (1) and (2) of reference (b) on a no cost to the Navy basis in connection with space piloting training. This suggestion is concurred in.

6. The Chief of Naval Personnel will explore other methods of near Earth navigation on a continuing basis as opportunity permits.

/s/ W. R. SMEDBERG, III

Copy to:
DCNO (AIR) (OP-54)
Captain P. V. H. Weems, USN (Ret)
CNATRA
COMTRAPAC

SPACE NAVIGATION

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	Preface
1	Introduction
2	The Universe
3	Solar System
4	Earth-Moon System
5	Position Finding
6	Space Dead Reckoning
7	Instrumentation for Space Navigation
8	Three Dimensional Mockups
9	Practical Problems in Space Navigation

Enclosure (1)

NAME	TITLE	ORGANIZATION
Captain P. V. H. Weems, USN (Ret) and Staff		Weems System of Navigation, Annapolis, Maryland
R. L. King	Asst. to Associate Administrator	Office of Associate Administrator, NASA
B. A. Mulcahy	Director of Technical Information	Information Division, NASA
George King	Head, Instructional Procedures and Special Programs Section (Curriculum and Instruction Branch)	Instructional Standards and Materials Division, BUPERS
W. A. Harrell	Head, Training Aids Section	PRNC
R. H. Deakin	Training Aids Branch Utilization Section	BUPERS
Herbert H. Rosen	Deputy Director, Office of Public Information	NASA
John F. Butcher	Training Aids Branch	BUPERS
L. Braaten	Training Aids Branch	BUPERS
Clotaire Wood	Asst. to Deputy Administrator	NASA
Lt. W. P. Vosseler, USN	Air Guided Missile Officer	Commander Training Command, STAFF U. S. Pacific Fleet San Diego 47, California
Captain Frank B. Corman, USN	Head, Naval Applications Branch, Astronautics Operations Division	CNO
CDR A. W. Belikow, USN	Head, Plans and Policy Branch, Astronautics Operations Division	CNO

ENCLOSURE (2)

SPACE NAVIGATION PRINCIPLE

Since the concept advanced on the other page of this sheet determines the observer's GP by observing the positions of known stars, the basic idea can perhaps best be shown by leaving it to the Space Navigator to observe the position of the Earth's center by any means he prefers. He could, for example, project the meridians and declination circles of a properly constructed and oriented transparent star half-globe against the Earth and against the star field, and then with his eye as a scanner and his brain as a computer, observe directly the SHA and Declination of the Earth's center. This position is that of the observer's Nadir, 180° from his Zenith, which is in the vertical, and therefore, determines the position of his GP in terms of SHA and Latitude (the same value as the Declination). SHA + GHA ♈ = Longitude, therefore, we have described how the observer's GP can be determined simply by observing the Earth's center against projected celestial meridians and declination circles.

Example: The space navigator observes the center of the earth to be at Lat. 39°11'S, SHA 245°05'. At the same time he measures the diameter of the earth to be 21°30'.2. Required, the SV's position.

Solution: Shift the position of the Earth's center 180°. This gives the GP of SV to be Lat. 39°11'N, SHA 65°05'. Had the time of observation been 2200 on 1 Jan. 1959, the GHA ♈ would have been for that instant 70°48', which, added to SHA of SV (65°05') would place the SV in Longitude 135°53'W.

From Table S-1, for 21°30'.2, the distance to GP from SV is found to be 15,000 miles.

Therefore, the SV's position is: 15,000 miles above Lat. 39°11'N, Long. 135°53'W.

28 October 1959 P. V. H. WEEMS

Pers-C114/99.9-blb
23 December 1960

From: Chief of Naval Personnel
To: Chief of Naval Operations

Subj: Exploratory Training in Space Navigation

Ref: (a) CNO ltr to CNP OP-762/orw, Ser: 24P76 dtd 10 Dec 1959 (NOTAL)
(b) CNP ltr to CNO Pers-C114-blb, Ser: C114/99.9 dtd 19 Feb 1960 (NOTAL)

1. Reference (a) requested the Chief of Naval Personnel to consult with Captain P. V. H. Weems, USN (Ret), owner of the Weems System of Navigation, and others as appropriate to ascertain the requirement for an exploratory development of space piloting training techniques.

2. Reference (b) indicated that orientational space piloting training is worthy of further pursuit, even though space navigational techniques are still in their infancy. The Navy would be well-advised to attempt some training in the area of space navigation to learn as much as possible about this new science.

3. Accordingly, it is planned to select three or four ensigns, graduates of NROTC (Contract or Regular) with appropriate majors (astronomy, physics, mathematics, or the like), to commence a training program after graduation this 1960-61 academic year. These officers would be ordered to report to the Superintendent, U. S. Naval Academy for temporary duty under instruction for four to six months. At Annapolis, they would work closely with Captain Weems in their studies. Captain Weems is prepared to participate as required. The officers involved would become the nucleus of trained and educated personnel potentially required to develop future courses in the subject area of space navigation. This could well be an opportunity for the Navy to take the lead in this field. Personnel records are being reviewed for selectees now.

4. Your comments and concurrence are requested.

/s/ W. R. SMEDBERG, III

Copy to:

DCNO (Air) (Op-54)
CAPT P.V.H. Weems, USN (Ret)
CNATRA
COMTRAPAC
Supt., USNA
Chief of Naval Research
Op 07

January 3, 1961

VICE ADMIRAL W. R. SMEDBERG, III,
CHIEF OF NAVAL PERSONNEL
ROOM 2072, NAVY DEPARTMENT ANNEX
ARLINGTON, VIRGINIA.

Dear Admiral Smedberg:

Pursuant to our conversation of 19 December 1960, I recommend the organization of a small Space Navigation Class of three to six young students under our personal supervision here at Annapolis. For best results, this class should operate on an intimate, informal basis with the students assisting in the development of the course from month to month, over a period of about six months.

We are in a position to furnish the immediately needed texts, Almanacs, Tables, reference library and demonstration equipment. Adequate living facilities are available at Annapolis.

As for the business arrangement for this operation, I suggest that either I be recalled to active duty for this period, and the government bear the cost of the class, or that a flat fee of $1,000 per student be allowed.

This plan would entail minimum cost, get the program started without delay, and under the auspices of the Navy Department, which should be to the national benefit by taking advantage of the Navy's experience in sea and air navigation. If considered proper, one or more students from NASA and from USAF could be invited to join the class.

In this venture mental work on the theory and practice of short-range space navigation would be stressed, with relatively little emphasis on hardware. Since about 1,000 pounds will be required to place in orbit one pound of payload, we should first investigate what can be done with the least possible equipment, thereby keeping both cost and weight to a minimum.

The proposed work would be based on the concept discussed with you, and mentioned in Chief of Naval Operation letter OP-762 Ser 24P76, 10 December 1959, to Chief of Naval Personnel, and your letter, Ser. C114/99.9 of 19 February 1960 to Chief of Naval Operations. I am firmly convinced that this program would promote national progress in the rapidly developing space age, and I, personally, would be highly honored to initiate it.

Sincerely,

/s/ P. V. H. WEEMS

PVHW:me

Pers-C114/99.9-pb
23 January 1961

From: Chief of Naval Personnel

To: Captain P. V. H. Weems, USN (Ret)
Randall House
Annapolis, Maryland

Subj: Space Navigation Class; initiation of

Ref: (a) Captain Weems ltr to CNP dtd 3 Jan 1961
(b) CNP ltr to CNO Pers-C114/99.9-blb dtd 23 Dec 1960
(c) CNP ltr to CNO (Op 762) Ser: C114/99.9 dtd 19 Feb 1960
(d) Telcon between Mr. George King (Pers-C114) and Captain P. V. H. Weems USN (Ret) of 12 Jan 1961

1. Reference (a) outlined a proposal for a Space Navigation Class to develop a course in this subject area. The Chief of Naval Personnel concurs in the proposal and agrees in principle with the concept expressed for conducting this exploratory Navy project. With regard to arrangements, the first suggestion, that of recall to active duty, is accepted. Steps are being taken to accomplish this. You will be ordered to report in an instructional status to the Superintendent, U. S. Naval Academy, for a period not to exceed six months, to initiate this project in accordance with guidelines set forth in reference (c) and reiterated in reference (a). Those NROTC graduates selected to participate will also report to the Superintendent, U. S. Naval Academy for temporary duty under instruction as outlined in reference (b). This endeavor is envisioned as an official U. S. Navy undertaking with the Navy to retain absolute control of results, findings, rights, etc., for their potential values to the U. S. Government without added or duplicated expense. Since the project is one of intensive exploratory study and research, it is assumed that no official travel will be involved other than that initially required to bring the student officers into Annapolis. By means of reference (d) it was determined that the only remaining cost consideration for the class is that of texts, reference materials, facilities and the like, already largely available in the Naval Establishment. By copy of this letter the Superintendent, U. S. Naval Academy, is requested to provide support with available facilities and equipment. The details of meeting other needs can be worked out by separate correspondence between now and class convening time shortly after this coming NROTC graduation.

2. In connection with proprietary data developed during your tour of duty with the Navy Department of this special project, the Navy shall obtain this without limitation as to its use. Of course, this does not extend to any data which originated before your return to active duty. As for copyrights, the U. S. Government policy is to reserve only a license under copyright on copyrighted data, leaving you free to copyright the material. The Navy also desires that no adverse claim of copyright be established in such data and that its right to reproduce and use such data shall be unlimited.

3. The desirability of inviting other services and agencies to participate in this project is recognized as a possibility later in the course after initial probings have narrowed the field somewhat. However, the preferred approach for now is to commence intensive study with a closely-knit, highly compatible Navy group in order to get to the heart of the matter with a minimum of orientation.

4. The Navy hopes to gain from this project a proposed officer course in Space Navigation and a delineation of recommended space piloting

Subj: Space Navigation Class; initiation of

training techniques. The task should prove to be a rewarding venture in this new science.

/s/ W. R. SMEDBERG, III

W. R. SMEDBERG, III

Copy to:

Supt., USNA
DCNO (Air) (Op-54)
CNO (Op-07)
CNATRA
COMTRAPAC
Chief of Naval Research

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